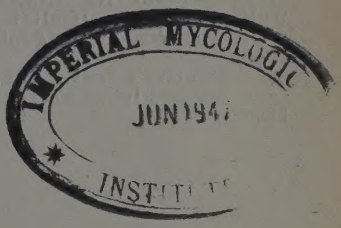
COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1945



VOLUME XLIX

№ 3

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours. Neuf numéros composent un volume pourvu d'une table des matières. Il y a quatre volumes par année.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» en de courtes communications précisent l'essentiel des travaux en préparation. Ces articles, ne dépassant pas en règle générale quatre pages, sont donnés, en anglais ou en français. Ils embrassent les sciences mathématiques, physiques, naturelles et appliquées, mettant au jour les résultats de recherches scientifiques en cours ou qui viennent d'être achevées, et représentant ainsi les plus récentes données de l'investigation scientifique.

L'auteur a droit à 50 exemplaires de son article.

Adressez les manuscrits à la Rédaction des «Comptes Rendus», Moscou, Wolkhonka, 14.

30 Octobre

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

D. Beliankin, de l'Académie, **V. Chlopin,** de l'Académie, **A. Frumkin,** de l'Académie, **A. Kolmogoroff,** de l'Académie (vice rédacteur), **L. Orbeli,** de l'Académie (vice rédacteur), **A. Richter,** de l'Académie, **S. Soboleff,** de l'Académie, **S. Vavilov,** de l'Académie (rédacteur en chef)

NOUVELLE SÉRIE

13-me année

1945

VOLUME XLIX, № 3

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

N. Dmitriev and **E. Dynkin.** On the Characteristic Numbers of a Stochastic Matrix 159
A. Kronrode. Sur les permutations des termes dans les séries numériques . . 163
Sh. Mikeladze. On Numerical Integration 166
A. G. Pinsker. On the Decomposition of *K*-spaces into Elementary Spaces . . 168

HYDROMÉCANIQUE

L. S. Leibenson, de l'Académie. Les nouvelles équations de mouvement d'un liquide saturé par un gaz dans un milieu poreux 172

PHYSIQUE MATHÉMATIQUE

G. P. Ivanzov. Résolution du problème de refroidissement d'un semi-espace rempli d'une matière dont les propriétés sont dépendantes de la température 175

PHYSIQUE

B. M. Wul, Corresponding Member of the Academy, and **I. M. Goldman.** Dielectric Constant of Barium Titanate as a Function of Strength of an Alternating Field 177
S. Orlov, Corresponding Member of the Academy. Energy Distribution in the Spectrum of a Black Radiator and Wien's Displacement Law 181

HYDROLOGIE

M. A. Velikanov, Corresponding Member of the Academy. Formation of Storm Floods 184

PHYSIQUE DU GLOBE

N. N. Kalitin. Illumination of the Earth Surface by Atmosphere Scattered Radiation, Ultra-violet, Green and Red, as Affected by the State of the Sky 188

Denoting this convex envelope by Q (it will be evidently a polygon with not more than n sides), we see that the polygon λQ lies inside Q.

2. Let, conversely, Q be a convex polygon with the vertices $z_1, z_2, \ldots, z_q$ such that λQ lies inside Q. Then we may choose such $p_{ij} \geqslant 0$ satisfying the equations $\sum_{j=1}^{n} p_{ij} = 1$ for $i = 1, 2, \ldots, q$, that

$$\lambda s_i = \sum_{j=1}^{q} p_{ij} z_j, \quad i = 1, 2, \ldots, q$$

To this end it suffices, for instance, to choose for every i the indices α, β, γ in such a way that the point λz_i should lie inside (or on the boundary) of the triangle $(z_\alpha z_\beta z_\gamma)$, and then put $p_{ij}=0$ for all $j \neq \alpha, \beta, \gamma$. Then the real numbers $p_{i\alpha}$, $p_{i\beta}$, $p_{i\gamma}$ will be uniquely determined by the equations

$$p_{i\alpha} z_\alpha + p_{i\beta} z_\beta + p_{i\gamma} z_\gamma = \lambda z_i$$
$$p_{i\alpha} + p_{i\beta} + p_{i\gamma} = 1$$

and will be non-negative. Consequently, λ is a characteristic number of a stochastic matrix $\| p_{ij} \|$ and the theorem is proved.

§ 3. We proceed to deduce from Theorem 1 the facts stated in the introduction.

In the first place, $|\lambda| \leqslant 1$. In fact, let A be one of the points of the polygon Q removed farthest from the origin. Since the point λA must lie inside Q, its distance from the origin must be less than or equal to the distance from the origin to the point A

$$|\lambda A| = |\lambda|\,|A| \leqslant |A|; \quad |\lambda| \leqslant 1$$

Let now $|\lambda| = 1$. It is evident that if A is a point of Q at the maximum distance from the origin, it must be a vertex of Q. But λA enjoys the same property, and consequently, is also a vertex of Q; the same holds for $\lambda^2 A$, $\lambda^3 A, \ldots$ *etc.*

Consequently, there are no more than n different points A, λA, $\lambda^2 A, \ldots$ Let their number be q, $q \leqslant n$. Then $\lambda^q A = A$, $\lambda^q = 1$, $\lambda = e^{2\pi i p/q}$ lies in the vertex of a regular q-angle with the centre at the origin and one of the vertices at the point 1. But such a regular q-angle being multiplied by $e^{2\pi i p/q}$ coincides with itself, and consequently, the number $e^{2i\pi p/q}$ may, indeed, be a characteristic number of a stochastic matrix.

It suffices to consider polygons Q containing the origin inside. In fact, supposing that $0 \in Q$ let B be a point of Q nearest to the origin. Evidently, there can exist only one such point. But then λB cannot lie in Q for $|\lambda| \leqslant 1$, except the trivial case $\lambda = 1$.

Hence we see that the domain of roots λ is star-like, *i. e.* along with every point λ it also contains all points of the segment $[0, \lambda]$, *i. e.* the points $\alpha\lambda$ where $0 \leqslant \alpha \leqslant 1$, since we have always $\alpha\lambda Q \subset \lambda Q \subset Q$.

§ 4. Let us prove a geometrical lemma.

Lemma. Denote in the polygon the angles $\angle z_2 z_1 0$, $\angle z_3 z_2 0, \ldots, \angle z_1 z_q 0$, respectively, by $\varphi_1, \varphi_2, \ldots, \varphi_q$. If Q is a regular q-angle with the centre in 0, then

$$\varphi_1 = \varphi_2 = \cdots = \varphi_q = \alpha = \frac{\pi}{2} - \frac{\pi}{q}$$

In the general case we shall always have

$$\min(\varphi_1, \varphi_2, \ldots, \varphi_q) \leqslant \alpha.$$

Proof. Suppose that $\varphi_1 > \alpha$, $\varphi_2 > \alpha$, ..., $\varphi_q > \alpha$. Consider the triangle $\triangle z_k z_{k+1} 0$ (Fig. 1). By assumption $\angle z_k > \alpha$. Let $\angle 0 z_k N_k = \alpha$; then $0 N_k < 0 z_{k+1}$. Denote the angle $\angle 0 N_k z_k$ by ψ_k. By the sine theorem

$$\frac{\sin \psi_k}{\sin \alpha} = \frac{0 z_k}{0 N_k} > \frac{0 z_k}{0 z_{k+1}}$$

Multiply such inequalities for $k = 1, 2, \ldots, q$. Then we obtain

$$\frac{\sin \psi_1}{\sin \alpha} \frac{\sin \psi_2}{\sin \alpha} \cdots \frac{\sin \psi_q}{\sin \alpha} > \frac{0 z_1}{0 z_2} \frac{0 z_2}{0 z_3} \cdots \frac{0 z_q}{0 z_1} = 1$$

On the other hand,

$$\sum_{k=1}^{q} \psi_k = q(\pi - \alpha) - \sum_{k=1}^{q} \angle z_{k+1} 0 z_k = q\left(\pi - \frac{\pi}{2} + \frac{\pi}{q}\right) - 2\pi = \frac{q\pi}{2} - \pi = q\alpha$$

But the product $\sin \psi_1 \sin \psi_2 \cdots \sin \psi_q$, in which the ψ_k satisfy the condition $\psi_1 + \psi_2 + \cdots + \psi_q = q\alpha$, attains in the domain $0 < \psi_1 < \pi$, ..., $0 < \psi_q < \pi$

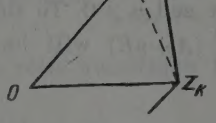


Fig. 1

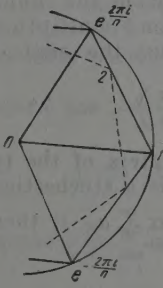


Fig. 2

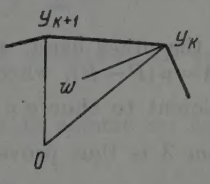


Fig. 3

its maximum for $\psi_1 = \psi_2 = \cdots = \psi_q = \alpha$, and so we obtain the desired contradiction.

Theorem 2. *The domain of characteristic numbers of stochastic matrices of the order not exceeding n in the sector* $-\frac{2\pi}{n} \leqslant \arg \lambda \leqslant \frac{2\pi}{n}$ *is bounded by two segments joining the point* 1 *with the points* $e^{2\pi i/n}$ *and* $e^{-2\pi i/n}$, *respectively.*

Proof. We first show that the indicated domain may in fact serve as the domain of roots. In virtue of the star-likeness it suffices to show that any point of the segments $[1, e^{2\pi i/n}]$ and $[1, e^{-2\pi i/n}]$ may serve as a root (Fig. 2). Let z be some point of the segment $[1, e^{2\pi i/n}]$. The regular n-angle P_n with the vertices $1, e^{2\pi i \frac{1}{n}}, e^{2\pi i \frac{2}{n}}, \ldots, e^{2\pi i \frac{n-1}{n}} = e^{-2\pi i \frac{1}{n}}$ being multiplied by z is transformed into a polygon zP_n, all vertices of which are situated on the sides of P_n; $zP_n \subset P_n$ and so z is a root.

Let us now show that, inversely, if z is a characteristic number of a stochastic matrix of the order n and $-2\pi/n \leqslant \arg z \leqslant 2\pi/n$, then z belongs to one of the triangles $10e^{2\pi i/n}$ or $10e^{-2\pi i/n}$ (Fig. 2).

Let Y be a convex polygon with the vertices $y_1, y_2, \ldots, y_q$, $q \leqslant n$, such that $zY \subset Y$. For the sake of definiteness suppose that $0 \leqslant \arg z \leqslant 2\pi/n$. Consider the angles $\angle y_2 y_1 0$, $\angle y_3 y_2 0$, ..., $\angle y_1 y_q 0$. According to the lemma, at least one of these angles will be $\leqslant \pi/2 - \pi/q \leqslant \pi/2 \leqslant \pi/n$. Let $\angle y_{k+1} y_k 0 \leqslant \pi/2 - \pi/n$.

The point $w = y_k z$ lies inside or on the boundary of the polygon Y, and consequently, $\angle w y_k 0 \leqslant \angle y_{k+1} y_k 0 \leqslant \frac{\pi}{2} - \frac{\pi}{n}$ (Fig. 3). But evidently $\angle w y_k 0 = \angle z\, 1\, 0$. Consequently, $\angle z\ 1\ 0 \leqslant \pi/2 - \pi/n$ and the point z lies inside the $\angle A_n\ 1\ 0$, where $A_n = e^{2\pi i/n}$, *q. e. d.*

§ 5. Theorem 3. *The domain of possible values λ' of characteristic numbers of matrices of order n $A = \|a_{ij}\|$ satisfying the conditions*

$$a_{ii} \leqslant 0,\ a_{ij} \geqslant 0,\ i \neq j, \sum_{j=1}^{n} a_{ij} = 0 \text{ for } i = 1,\ 2, \ldots,\ n,$$

is the angle with the ver-tex in the origin

$$\pi = \left(\frac{\pi}{2} - \frac{\pi}{n}\right) \leqslant \arg \lambda' \leqslant \pi + \left(\frac{\pi}{2} - \frac{\pi}{n}\right)$$

Remark. Such conditions are satisfied by matrices of the probability densities of the transition from the i-th state into the j-th state for Markoff's chains with a finite number n of states and a continuous time.

Proof. Let P be an arbitrary stochastic matrix, and μ, an arbitrary positive number. The matrix $A = \mu(P - E)$ satisfies our conditions; its characteristic numbers λ' are obtained from the characteristic numbers λ of the matrix P by the formula $\lambda' = \mu(\lambda - 1)$. Hence the domain of values of λ' contains the figure obtained from the domain of values of λ by a translation to the left by one unit and multiplication by μ. The sum of all such figures for all possible μ is just the angle

$$\pi - \left(\frac{\pi}{2} - \frac{\pi}{n}\right) \leqslant \arg \lambda' \leqslant \pi + \left(\frac{\pi}{2} - \frac{\pi}{n}\right)$$

On the other hand, any matrix of the type A may be represented in the form $A = \mu(P - E)$, where P is a stochastic matrix and $\mu > 0$. To this end it is sufficient to choose $\mu > \max\limits_{1 \leqslant i \leqslant n} (|a_{ii}|)$; then $P = \frac{1}{\mu}(A + \mu E)$ will be proved. Theorem 3 is thus proved.

The M. V. Lomonosov State University.
Moscow.

Received
5. VII. 1945.

REFERENCES

[1] V. Romanovsky, Acta Math., 66, 147 (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

MATHÉMATIQUES

SUR LES PERMUTATIONS DES TERMES DANS LES SÉRIES NUMÉRIQUES

Par A. KRONRODE

(Présenté par A. N. Kolmogoroff, de l'Académie, le 17. 1. 1945)

Le présent article a pour origine la solution du problème suivant posé par M. Kreines. Si une permutation P effectuée sur une série numérique α convergente la transforme en une série encore convergente, mais dont la somme est différente, existe-t-il toujours une série convergente β telle que la permutation P la transforme en une série divergente?

Une permutation P des termes d'une série infinie $\sum_{k=1}^{\infty} a_k$ correspondant à la substitution $\begin{pmatrix} a_1 & a_2 & \dots & a_k & \dots \\ a_{n_1} & a_{n_2} & \dots & a_{n_k} & \dots \end{pmatrix}$ est nommée neutre, si quelle que soit la série numérique convergente $\alpha \equiv \sum_{i=1}^{\infty} a_i$ dont la somme est égale à A, la série $P\alpha \equiv \sum_{i=1}^{\infty} a_{n_i}$ est encore convergente et de somme égale à A, et si quelle que soit la série divergente $\beta = \sum_{i=1}^{\infty} b_i$, la série $P\beta \equiv \sum_{i=1}^{\infty} b_{n_i}$ est encore divergente.

Une permutation P est nommée permutation gauche s'il existe une série convergente α telle que la série $P\alpha$ diverge. Une permutation inverse à une permutation gauche est nommée permutation droite.

Si une permutation gauche (droite) n'est pas une permutation droite (resp. gauche), elle est nommée gauche unilatérale (resp. droite unilatérale). Une permutation qui est droite et gauche simultanément sera nommée bilatérale.

Si pour une permutation P il existe une série convergente α de somme A telle que la série $P\alpha$ converge vers un nombre fini $A_1 \neq A$, la permutation est nommée essentielle.

Déterminons pour chaque permutation P et pour chaque entier positif N une fonction à valeurs entières $\Lambda_P(N)$ par les conditions: Soient $l_1 < l_2 < \dots < l_r \leqslant N$ tous les entiers positifs tels que $P(l_i) > N$. Soit $\Lambda_P(N)$ le nombre de segments de la suite de nombres entiers positifs non empiétant et non tangents dont la somme coïncide avec l'ensemble des l_i. Soit $\overline{\Lambda}_P(N)$ le nombre de segments non empiétant et non tangents dont la somme coïn-

cide avec l'ensemble de tous les entiers m_i $(i=1, 2, \ldots, s)$ tels que $N < m_1 < m_2 < \ldots m_s$ et $P(m_i) \leqslant N$. Posons

$$\Lambda_P(N) = \Delta_P(N) + \Lambda_P(N).$$

Théorème 1. *Pour qu'une permutation P soit gauche, il faut et il suffit que l'on ait*

$$\overline{\lim_{N\to\infty}} \Delta_P(N) = \infty$$

Posons

$$\Pi_P(N) = \Lambda_{P^{-1}}(N).$$

Théorème 2. *Pour qu'une permutation P soit droite, il faut et il suffit que l'on ait*

$$\overline{\lim_{N\to\infty}} \Pi_P(N) = \infty$$

Théorème 3. *Chaque permutation essentielle est bilatérale.*

Il existe des permutations unilatérales et des permutations bilatérales qui ne sont pas essentielles.

Théorème 4. *Le produit de deux permutations non gauches (non droites) est une permutation non gauche (resp. non droite).*

Théorème 5. *Chaque permutation P peut être représentée dans la forme $P = P_1 \cdot P_2$, où P_2 est non droite et P_1 est non essentielle.*

Corollaire. Chaque série non absolument convergente peut être transformée en une série divergente au moyen d'une permutation gauche unilatérale.

Théorème 6. *Pour que la permutation P soit essentielle, il faut et il suffit que l'on ait*

$$\lim_{N\to\infty} \min[\Delta_P(N), \Pi_P(N)] = \infty$$

Définition. Nous disons qu'une série dont les termes sont des nombres réels, converge vers $+\infty$, si quel que soit $K > 0$ il existe un entier N_k tel que $\sum^{T} \alpha > K$ pour $T > N_k$, et il existe une constante $W > 0$ telle que pour toutes les valeurs de L et M, $M > L$, $\sum_{L}^{M} \alpha > -W$ (je désigne par $\sum^{S} \alpha$ la S-ième somme partielle de la série α, et je pose $\sum_{T}^{U} \alpha \equiv \sum^{U} \alpha - \sum^{T} \alpha$ pour $U > T$).

Je dis que la série α converge vers $-\infty$, si la série $(-\alpha)$ que l'on obtient en changeant les signes de tous les termes de la série α converge vers $+\infty$.

Théorème 7. *Une permutation droite unilatérale transforme une série à termes réels qui converge vers $+\infty$ $(-\infty)$ en une série qui converge encore vers $+\infty$ (resp. $-\infty$).*

Définition. Une série $\alpha = \sum_{i=1}^{\infty} a_i$ à termes réels est nommée non absolument divergente, si la série α ne converge pas (en particulier ne converge ni vers $+\infty$, ni vers $-\infty$), et si d'ailleurs $\lim_{i\to\infty} a_i = 0$.

Théorème 8. *Chaque série à termes réels qui n'est pas absolument divergente peut être transformée en une série qui converge vers une somme quelconque finie S, et cela au moyen d'une permutation non essentielle P; d'ailleurs, cette permutation P peut être choisie de telle manière qu'elle puisse être repré-*

sentée dans la forme $P = P_1 \cdot P_2$, *où* P_1 *est une permutation non gauche et* P_2 *une permutation non droite, et d'ailleurs* $\lim_{n\to\infty} \Lambda_P(n) \neq \infty$.

Théorème 9. *Chaque série* α *à termes réels et non absolument convergente peut être transformée en une série qui converge vers une somme* S *arbitrairement donnée au moyen d'une permutation* P *qui peut être représentée dans la forme* $P = P_1 \cdot P_2$, *où* P_1 *est une permutation droite unilatérale et* P_2 *une permutation gauche unilatérale.*

Pour les séries à termes complexes on a des théorèmes analogues aux théorèmes 8 et 9.

Définition. Une série divergente $\alpha \equiv \sum_{i=1}^{\infty} a_i$ à termes complexes est nommée non absolument divergente si l'on a $\lim_{i\to\infty} a_i = 0$.

Théorème 10. *Soit* α *une série à termes complexes non absolument divergente et vérifiant la condition* $\lim_{n\to\infty} \left| \sum^{n} \alpha \right| \neq \infty$. *Alors:*

1°. *Il existe une droite AB telle que pour chaque point S de cette droite il existe une permutation P vérifiant les conditions du théorème 8 et telle que la série* $P\alpha$ *converge vers S.*

2°. *Si dans un système rectangulaire de coordonnées* (x, y) *tel que l'axe y coïncide avec AB on a* $\overline{\lim_{n\to\infty}} \left| x\left(\sum^{n} \alpha\right) \right| \neq 0$ *(où l'on désigne par* $x(a)$ *l'abscisse x du vecteur a), alors pour chaque point du plan il existe une permutation P vérifiant les conditions du théorème 8 et telle que la série* $P\alpha$ *converge vers ce point.*

3°. *Si* $\lim_{n\to\infty} \left| x\left(\sum^{n} \alpha\right) \right| = 0$, *la permutation P, vérifiant la condition* $\lim_{n\to\infty} \min[\Lambda_P(n), \Pi_P(n)] \neq \infty$, *ne peut pas transformer la série* α *en une série qui converge vers un point situé hors de la droite AB.*

Théorème 11. *Soit* $\alpha \equiv \sum_{i=1}^{\infty} a_i$ *une série non absolument convergente à termes complexes. Alors:*

1°. *Pour chaque point S d'une certaine droite AB il existe une permutation* $P = P_1 \cdot P_2$ *telle que la série* $P\alpha$ *converge vers S, et d'ailleurs* P_1 *est une permutation unilatérale droite et* P_2 *une permutation unilatérale gauche.*

2°. *Il passe par le point qui correspond à la somme de la série* α *au moins une droite vérifiant la condition* 1°.

3°. *S'il existe une permutation R de nature arbitraire qui transforme la série* α *en une série* $R\alpha$ *telle qu'un point C situé hors de la droite AB est un point limite de la suite* $\sum^{n} \cdot R\alpha$ $(n = 1, 2, \ldots)$, *alors la série* α *peut être transformée au moyen d'une permutation P vérifiant les conditions de* 1° *en une série qui converge vers un point du plan arbitraire donné.*

Ce dernier théorème est en quelque sorte une précision d'un théorème de Lévy-Steinitz [1] (pour le cas où $n = 2$); d'après ce théorème les permutations des termes dans une suite non absolument convergente de vecteurs dans l'espace à n dimensions transforment cette suite en des suites qui convergent vers tous les points d'un certain sous-espace linéaire de l'espace donné.

Manuscrit reçu
le 17. I. 1945.

LITTÉRATURE CITÉE

[1] E. Steinitz, J. f. reine und angew. Mathem., **143**, 128 (1913).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

MATHEMATICS

ON NUMERICAL INTEGRATION

By Sh. MIKELADZE

(Communicated by A. N. Kolmogoroff, Member of the Academy, 9. XII. 1944)

In the formulae of numerical integration derived hereafter only the values of the integrated function and its first derivative are used. One of the results gives a formula discussed by Frame [1].

Consider the integral

$$\int_{a-mh}^{a+mh} f(x)\,dx = h\int_0^m [f(a+th)+f(a-th)]\,dt \tag{1}$$

Let $t_0=0,\ \pm t_1, \ldots, \pm t_r$ denote different values of t. It is easy to show that

$$f(a+th)+f(a-th) = \sum_{\nu=0}^{r} \frac{P_\nu^2(t)}{P_\nu^2(t_\nu)}\,[f(a+t_\nu h)+f(a-t_\nu h)] +$$

$$+\frac{h}{2}\sum_{\nu=1}^{r}\frac{P_\nu(t)\,P(t)}{t_\nu P_\nu^2(t_\nu)}\left\{[f'(a+t_\nu h)-f'(a-t_\nu h)]-\right.$$

$$\left.-\frac{2}{h}\,\frac{P_\nu'(t_\nu)}{P_\nu(t_\nu)}\,[f(a+t_\nu h)+f'(a-t_\nu h)]\right\}+R \tag{2}$$

where

$$R = 2h^{4r+2}P^2(t)\,\frac{f^{(4r+2)}(a+\vartheta h)}{(4r+2)!}$$

and

$$P(t) = t(t^2-t_1^2)\ldots(t^2-t_r^2)$$

$$P_\nu(t) = t(t^2-t_1^2)\ldots(t^2-t_{\nu-1}^2)(t^2-t_{\nu+1}^2)\ldots(t^2-t_r^2)$$

On substituting in (1) the expression (2), we get the formula

$$\int_{a-mh}^{a+mh} f(x)\,dx = 2hA_0 f(a)+h\sum_{\nu=1}^{r}A_\nu[f(a+t_\nu h)+f(a-t_\nu h)]+$$

$$+h^2\sum_{\nu=1}^{r}D_\nu[f'(a+t_\nu h)-f'(a-t_\nu h)]$$

$$A_\nu = \int_0^m \frac{P_\nu^2(t)}{P_\nu^2(t_\nu)}\,dt - \frac{P_\nu'(t_\nu)}{P_\nu(t_\nu)}\,D_\nu,\quad D_\nu = \frac{1}{2}\int_0^m \frac{P_\nu(t)\,P(t)}{t_\nu P_\nu^2(t_\nu)}\,dt, \tag{3}$$

with the residual term

$$2h^{4r+3}\frac{f^{(4r+2)}(a+\vartheta h)}{(4r+2)!}\int_0^m P^2(t)\,dt$$

ϑ being included in an interval containing $0, \pm t_1, \ldots, \pm t_r$ and $\pm m$.

Divide the whole interval of integration into an arbitrary number of equal subintervals, and use for each subinterval formula (3) with $m=1$, $r=1$ and $t_1=1$. On summing up the expression so obtained we get the formula deduced by Frame [1].

Now consider the integral

$$\int_{a-mh}^{a+mh} f(x)\,dx = 2mhf(a)+h^2\int_0^m (m-t)\,[f'(a+th)-f'(a-th)]\,dt$$

and let us substitute for $f'(a+th)-f'(a-th)$ the expression

$$f'(a+th)-f'(a-th)=\sum_{\nu=1}^{r}\frac{P_\nu(t)}{P_\nu(t_\nu)}\,[f'(a+t_\nu h)-f'(a-t_\nu h)]+$$
$$+2h^{2r+1}P(t)\,\frac{f^{(2r+2)}(a+\vartheta h)}{(2r+1)!}$$

which can be obtained with the help of Lagrange's interpolation formula. Then

$$\int_{a-mh}^{a+mh} f(x)\,dx = 2mhf(a)+h^2\sum_{\nu=1}^{r} D_\nu\,[f'(a+\nu h)-f'(a-\nu h)]+R$$

Here

$$D_\nu=\int_0^m (m-t)\,\frac{P_\nu(t)}{P_\nu(t_\nu)}\,dt$$

$$R=\frac{2h^{2r+3}}{(2r+1)!}\int_0^m (m-t)\,P(t)\,f^{(2r+2)}(a+\vartheta h)\,dt$$

ϑ being included in an interval containing $0, \pm t_1, \ldots, \pm t_2$ and $\pm m$.

The particular case $m=1$, $r=1$ and $t_1=1$ is worthy of notice.

Mathematical Institute.
Academy of Sciences of the Georgian SSR.

Received
6. IX. 1944.

REFERENCES

[1] J. S. Frame, The Amer. Math. Monthly, **50**, No. 4, 244 (1943).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

MATHEMATICS

ON THE DECOMPOSITION OF *K*-SPACES INTO ELEMENTARY SPACES

By A. G. PINSKER

(Communicated by A. N. Kolmogoroff, Member of the Academy, 9. IX. 1944)

The structure of an arbitrary K-space was discussed in the preceding paper([1]). The main result obtained there is that every K-space is contained in a certain «universal» space which is an extension of the Kantorovitch-Banach's space (of the type KB_5). Starting from this proposition it is possible to solve the following problem of the general theory of K-spaces:

To decompose the K-space into spaces of a simpler nature, and so to reduce the study of the K-spaces of the general kind to the study of these simplest spaces.

In the present paper we introduce the notion of an elementary space, consider certain properties of such spaces and establish the possibility of representation of an arbitrary K-space in the form of a disjunctive sum of elementary spaces. The definitions and the notations of the paper([1]) are supposed to be known to the reader.

1°. Theorem 1. *Let X be a K-space and x^* an arbitrary element of X different from zero. The aggregate of elements $x \in X$ satisfying the conditions:* α) $\inf(|x|, |x^*|) > 0$ *and* β) *if* $0 < |x'| < |x|$, *then* $\inf(|x'|, |x^*|) > 0$ *forms together with the zero a normal subspace $X^* \subset X$, a space generated by the element x^* and such that* 1) *X^* is a space with a unit;* 2) *every element $x \in X$ is uniquely representable in the form $x = x' + x''$, where $x' \in X^*$ and x'' is disjunctive to X^*, x' is the component of the element x (the projection of x) in the space X^*;* 3) *if the elements x_1^* and x_2^* are disjunctive, then the spaces generated by them are also disjunctive; and* 4) *if the set $E \subset X^*$ is bounded in X, then its exact bounds* $\sup E$ *and* $\inf E$ *belong to X^*.*

Theorem 2. *Whatever be a complete set $\{x_\xi\}$ of pairwise disjunctive and different from zero elements of a K-space X, X may be decomposed into a disjunctive sum of spaces X_ξ generated by the elements x_ξ so that every element $x \in X$, $x > 0$, is uniquely representable in the form $x = \sup\{x'_\xi\}$, where x'_ξ is the component of x in the space X_ξ.*

Example. The K-space $V(a, b)$ (of functions of bounded variation defined in the interval (a, b)) can be represented in the form of a normed disjunctive sum of K-spaces $L_\xi(a, b)$ (of summable functions defined in (a, b)) and of l_ξ (numerical sequences $\{\xi_n\}$ such that $\sum_{n=1}^{\infty} |\xi_n| < +\infty$),

$$V = \sum_\xi L_\xi + \sum_\xi l_\xi$$

this relation enables us to obtain easily the general forms of different functionals and operations in the space.

The decomposition into spaces with unit may be, however, carried out in such a way that each component should represent, in a certain sense, a simplest space. To this end we introduce the following definition:

A K-space is said to be elementary, if 1) it contains a unit, and 2) if its extension is a space of the type R_4.

The spaces of the type R_4 were considered by Kantorovitch [2]; in such spaces a real metrical function satisfying certain conditions is defined for element $x \geqslant 0$.

The spaces $S(a, b)$ (of measurable functions defined in the interval (a, b)) and s (of all possible numerical sequences) are important examples of spaces of the type R_4 and of elementary spaces.

Theorem 3. a) *An extension of an elementary space is an elementary space.*

b) *The space generated by an element of an elementary space is an elementary space.*

c) *A subspace with a unit of an elementary space is an elementary space.*

d) *The disjunctive sum of a denumerable set of elementary spaces is an elementary space.*

Theorem 4. *Each of the following conditions is necessary and sufficient for X, a K-space with a unit, to be elementary:*

1) *There exists in X not more than a denumerable set of pairwise disjunctive elements.*

2) *Every monotonic transfinite sequence of elements of X contains not more than a denumerable set of different elements.*

3) *Whatever be the set $E \subset X$, there exists a denumerable subset $E' \subset E$ such that* $\sup E' = \sup E$.

4) *If the set of elements $x_n^{(k)}$ $(n = 1, 2, \ldots; k = 1, 2, \ldots)$ is bounded, $x_n^{(k)} \to x_n$ for $k \to \infty$ and $x_n \to x$ for $n \to \infty$, then there exists a sequence of indices k_n such that $x_n^{(k_n)} \to x$ for $n \to \infty$.*

5) *For positive elements of X there may be determined a real function $\rho(x)$ enjoying the property: $\rho(x_1) > \rho(x_2)$, if $x_1 > x_2$.*

Theorem 5. *Whatever be an element x of the K-space X, there exists an element $x^* \leqslant |x|$ such that the space X^* generated by the element x^* is elementary.*

This theorem which follows from the results of my preceding note, implies the following fundamental proposition.

Theorem 6. *Every K-space may be represented in the form of a disjunctive sum of elementary spaces.*

2°. We proceed with the consideration of elementary spaces.

Theorem 7. *Each of the following conditions is sufficient for X, a K-space with a unit, to be elementary:*

1) *X is a regular space (of the type K_σ), i. e. in X Kantorovitch's axiom of regularity is satisfied.*

2) *X is an almost regular space.*

3) *X is a normed space of the type KB_2.*

Some interesting sufficient conditions of elementarity of spaces can be established by means of the continuum postulate (or, strictly speaking, by means of certain corollaries of this postulate).

Theorem 8. *Each of the following conditions is sufficient for X, a K-space with a unit, to be elementary:*

1) *In X holds the convergence with a regulator, i. e. if $x_n \to x\,(0)$, then there exists an element x_0 such that whatever be $\varepsilon > 0$ for all $n \geqslant N_\varepsilon$*

$$|x_n - x| \leqslant \varepsilon x_0$$

2) *Whatever be a sequence $\{x_n\}$ of elements of X, there exists such a sequence of real numbers λ_n that $\lambda_n x_n \to 0\,(0)$ for $n \to \infty$.*

3) *X is a separable space, i. e. X contains a denumerable set dense everywhere.*

We state yet another criterion of elementarity of a space.

Theorem 9. *A K-space with a unit, in which for the unit element is determined a real and essentially positive and additive function, i. e. a function* $\varphi(e)$ *satisfying the conditions:* 1) $\varphi(e) > 0$ *for* $e < 0$ *and* 2) $\varphi(e_1 + e_2) = \varphi(e_1) + \varphi(e_2)$, *if* $\inf(e_1, e_2) = 0$, *is an elementary space.*

Observe that the function $\varphi(e)$ is not supposed to be continuous in any sense whatever.

The convergence in elementary spaces enjoys properties similar to the known properties of convergence of measurable functions.

Theorem 10. *If X is an elementary space, then* a) *the relation* $x_n \to x\,(0)$ *is equivalent to the following one:* $|x_n - x| \leqslant \varepsilon(1 - e_n) + x^*_{e_n}$, *where* $\varepsilon > 0$ *is arbitrarily small,* $n \geqslant N_\varepsilon$, $e_n \to 0\,(0)$, $x^* = \sup_n \{x_n\}$ *and* $x^*_{e_n}$ *is the component of* x^* *in the space generated by the element* e_n, b) *the relation* $x_n \to x\,(*)$ *in the extension of X is equivalent to the following one: whatever be a positive number* λ, $\lim_{n\to\infty} \rho[e_\lambda(|x_n - x|)] = 0$, *where* e_λ *is the unit element introduced by H. Freudenthal and* ρ, *the metrical function, the existence of which is postulated in the definition of an elementary space.*

Thus, the topological convergence in an extension of an elementary space is a «convergence by measure».

3°. The propositions concerning elementary spaces stated above establish a far reaching analogy between the properties of elementary spaces, on the one hand, and the properties of spaces of measurable functions and numerical sequences, on the other. The analogy may be extended further still, if the operation of multiplication of elements is introduced in *K*-spaces. Wulich [3] gave an original definition of multiplication in *K*-spaces with a unit, yet the following definition of this operation seems more natural and simple.

Let X be an arbitrary K-space; $\tilde{X}$, its extension, which is, consequently, a space with a unit; and $E = \{e\}$, the aggregate of its unit elements. To $\tilde{X}$ belong all possible «step» elements of the form $\sum_{i=1}^{\infty} \lambda_i e_i$, where the e_i are pairwise disjunctive, $\sum_{i=1}^{\infty} e_i = 1$ and the λ_i are arbitrary real numbers. We define in the first line the multiplication of step elements: if $x = \sum_{i=1}^{\infty} \mu_i e'_i$ and $y = \sum_{i=1}^{\infty} \lambda_i e''_i$, then their product $xy = \sum_{i,j} \mu_i \lambda_j \inf(e'_i, e''_j)$ $(i, j = 1, 2, \ldots)$; it is easily seen that xy is also a step element. Let now x and y be arbitrary positive elements of X; A, the set of positive step elements $\leqslant x$; $\overline{A}$, the set of step elements $\geqslant x$, and denote by B and $\overline{B}$ the corresponding sets for the element y; all possible products $x_\xi y_\eta$, where $x_\xi \in A$, $y_\eta \in B$, form the set C, and C is defined similarly. It is evident that no element of C exceeds any element of $\overline{C}$, and so $\sup C \leqslant \inf \overline{C}$. On the other hand, for every $\delta > 0$ there exist elements $x' \in A$, $x'' \in \overline{A}$, $y' \in B$ and $y'' \in \overline{B}$ such that $x' \leqslant x \leqslant x''$, $y' \leqslant y \leqslant y''$, $x'' - x' \leqslant \delta \cdot 1$, and $y'' - y' \leqslant \delta \cdot 1$; hence we easily conclude that

$$x''y'' - x'y' \leqslant (x' + y')\delta \cdot 1 + \delta^2 \cdot 1 \leqslant \delta(x + y + 1),$$

and consequently, in view of the arbitrariness of δ, that $\sup C = \inf \overline{C}$. But, by definition, $xy = \sup C = \inf \overline{C}$. In the general case, when $x = x_+ - x_-$, $y = y_+ - y_-$, we put $xy = x_+y_+ - x_+y_- - x_-y_+ + x_-y_-$.

The so defined product of elements in $\tilde{X}$ enjoys the usual properties: it is associative, commutative and distributative; in particular, for every $x \in \tilde{X}$, $x \cdot 1 = x$, $x^2 = |x|^2$, the relation $xy = 0$ is equivalent to $\inf(|x|, |y|) = 0$, *etc.*

Let the set of unit elements of a K-space with a unit be called its basis. The bases of two K-spaces with a unit are isomorphic, if it is possible to establish between their elements a one-to-one correspondence preserving the relations of order.

Theorem 11. *In the extension of every elementary space $\tilde{X}$ is contained a normal subspace $L(\tilde{X})$ of the type KB_5 with a basis isomorphic to that of X. All normal subspaces of $\tilde{X}$ of the type KB_5 are isometric to $L(\tilde{X})$.*

Theorem 12. *All subspaces of the type KB_2 of an extension of an elementary space $\tilde{X}$ are normally contained in $L(\tilde{X})$.*

Choose in $\tilde{X}$ the set of elements $L^p(\tilde{X})$ $(p > 0)$ by putting $x \in L^p(\tilde{X})$ if $x^p \in L(\tilde{X})$. It is easily seen that $L^p(\tilde{X})$ is a normal subspace of $\tilde{X}$ and, moreover, of the type KB_4 if $p \geqslant 1$. In particular we note the case $p = 2$.

Theorem 13. *In $L^2(\tilde{X})$ we may define a scalar product of elements in such a way that $L^2(\tilde{X})$ becomes a (real) Hilbert space.*

It is also possible to introduce into consideration the spaces $L_T(\tilde{X})$ similar to the space L_T of measurable functions considered by L. Kantorovitch.

The Buryat-Mongol Pedagogical Institute.
Ulan-Udé.

Received
9.VIII.1944.

REFERENCES

[1] A. G. Pinsker. C. R. Acad. Sci. URSS, XLIX. No. 1 (1945). [2] L. V. Kantorovitch, Rec. Math., 2 (44), 121 (1937). [3] B. Z. Wulich, C. R. Acad. Sci. URSS, XXVI, No. 9 (1940).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

HYDROMÉCANIQUE

LES NOUVELLES ÉQUATIONS DE MOUVEMENT D'UN LIQUIDE SATURÉ PAR UN GAZ DANS UN MILIEU POREUX

Par L. S. LEIBENSON, de l'Académie

Le problème du mouvement d'un liquide saturé par un gaz dans le milieu poreux a été posé en premier par nous et analysé dans nos travaux [1] de 1934 et de 1941, en supposant que les bulles de gaz soient entraînées par le courant du liquide. En 1936 les savants américains Muskat et Meres ont envisagé un cas où les vitesses du gaz et du liquide étaient différentes. Pour la détermination des coefficients de perméabilité, distinctement pour le gaz et pour le liquide, ont été utilisées des données expérimentales [3] de Wyckoff et de Botset.

Nous donnons dans le suivant les nouvelles équations de mouvement, basées sur les considérations de l'hydrodynamique, en admettant que les vitesses du liquide et du gaz soient différentes.

Soient données les coordonnées rectangulaires x, y, z. Désignons par u_1, v_1, w_1, les composantes de vitesse du liquide, la densité par q, le coefficient de la viscosité absolue par μ_1, le coefficient de perméabilité dans un milieu homogène par k, la pression hydrodynamique par p. De même pour le gaz l'on désigne sa densité par ρ, le coefficient de viscosité absolue par μ_2, les composantes de vitesse par u_2, v_2, w_2.

La grandeur q, étant la densité du liquide dissolvant le gaz, est par suite une certaine fonction de la pression p. De plus, le gaz est en état de bulles, et suivant que la pression diminue, la quantité de ce gaz libre augmente, par suite du dégagement du gaz de la solution. Désignons la part du liquide dans une unité de volume par σ et appelons-la saturation [2].

Appliquons une règle connue de Darcy pour la détermination de la vitesse de filtration du liquide et du gaz. L'on a [2]:

$$\left.\begin{aligned} u_1 &= -\frac{k}{\mu_1} f \frac{\partial p}{\partial x}, \quad u_2 = -\frac{k}{\mu_2} \varphi \frac{\partial p}{\partial x} \\ v_1 &= -\frac{k}{\mu_1} f \frac{\partial p}{\partial y}, \quad v_2 = -\frac{k}{\mu_2} \varphi \frac{\partial p}{\partial y} \\ w_1 &= -\frac{k}{\mu_1} f \frac{\partial p}{\partial z}, \quad w_2 = -\frac{k}{\mu_2} \varphi \frac{\partial p}{\partial z} \end{aligned}\right\} \qquad (1)$$

Nous avons introduit là deux coefficients $f = f(\sigma)$, $\varphi = \varphi(\sigma)$, définis au moyen des graphiques obtenus à la base des données expérimentales de Wyckoff et Botset [3]. Il est évident que l'on a

$$f(0) = 0, \quad f(1) = 1, \quad \varphi(0) = 1, \quad \varphi(1) = 0 \qquad (1')$$

L'étude des fonctions $f(\sigma)$ et $\varphi(\sigma)$ a été déjà donnée dans notre autre

travail (4). Afin de déterminer deux inconnues $p\ (x, y, z, t)$ et $\sigma(x, y, z, t)$ nous allons partir de l'équation de continuité, séparément pour le liquide et le gaz. Etant donné que, lorsque la pression du liquide diminue, a lieu le dégagement du gaz dissout, et que le changement correspondant de densité q est insignifiant, nous supposons, pour simplifier, que cette dernière constante soit égale à q_0. La masse du gaz dégagée de la solution est définie par la loi de Henry. On trouve pou rle courant du liquide l'équation de continuité de la forme

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}-\frac{mc\sigma}{q_0}\frac{Dp}{Dt}+m\frac{\partial\sigma}{\partial t}=0 \tag{2}$$

où m est la porosité et c le coefficient de loi de Henry, que nous supposons, pour simplifier, être constant.

La masse du gaz dégagée de la solution va s'ajouter au gaz libre se mouvant dans le milieu poreux. Pour cela l'équation de continuité s'exprimera pour le gaz sous la forme

$$\frac{\partial(\rho u_2)}{\partial x}+\frac{\partial(\rho v_2)}{\partial y}+\frac{\partial(\rho w_2)}{\partial z}+mc\sigma\frac{Dp}{Dt}+m\frac{\partial}{\partial t}[\rho(1-\sigma)]=0 \tag{3}$$

où l'on a

$$\frac{Dp}{Dt}=\frac{\partial p}{\partial t}+\frac{\partial p}{\partial x}u_1+\frac{\partial p}{\partial y}v_1+\frac{\partial p}{\partial z}w_1 \tag{4}$$

En substituant (1) dans (2) l'on obtient

$$f(\sigma)\nabla^2 p+f'(\sigma)\Delta(p,\sigma)+\frac{\mu_1 mc\sigma}{kq_0}\left[\frac{\partial p}{\partial t}-\frac{k}{\mu_1}f(\sigma)\Delta_1 p\right]=\frac{m\mu_1}{k}\frac{\partial\sigma}{\partial t} \tag{5}$$

où

$$\nabla^2 p=\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2} \tag{6}$$

$$\Delta(p,\sigma)=\frac{\partial p}{\partial x}\frac{\partial\sigma}{\partial x}+\frac{\partial p}{\partial y}\frac{\partial\sigma}{\partial y}+\frac{\partial p}{\partial z}\frac{\partial\sigma}{\partial z} \tag{7}$$

$$\Delta_1 p=\left(\frac{\partial p}{\partial x}\right)^2+\left(\frac{\partial p}{\partial y}\right)^2+\left(\frac{\partial p}{\partial z}\right)^2 \tag{8}$$

Admettons pour la mesure de l'état du gaz l'hypothèse isothermique, qui est en concordance suffisante avec l'expérience. L'on a

$$p=\beta\rho \tag{9}$$

où β est la constante du gaz.

En substituant (1) et (9) dans (3) nous obtenons

$$\varphi(\sigma)(p\nabla^2 p+\Delta_1 p)+p\varphi'(\sigma)\Delta(p,\sigma)-\frac{m\beta c\mu^2\sigma}{k}\left[\frac{\partial p}{\partial t}-\frac{k}{\mu_1}f(\sigma)\Delta_1 p\right]=$$
$$=\frac{m\mu_2}{k}\frac{\partial}{\partial t}[p(1-\sigma)] \tag{10}$$

Les équations non linéaires aux dérivées partielles du second ordre (5) et (10) avec les conditions initiales et limites servent à la détermination de deux inconnues

$$p(x, y, z, t)\ \text{et}\ \sigma(x, y, z, t)$$

Au point de vue physique la saturation σ ne doit dépendre que de la pression p. Mais la forme de cette dépendance doit être définie des équations (5) et (10). En supposant

$$\sigma=\sigma(p) \tag{11}$$

on a les relations

$$\frac{\partial \sigma}{\partial t} = \frac{d\sigma}{dp}\frac{\partial p}{\partial t}, \quad \Delta(p, \sigma) = \frac{d\sigma}{dp}\Delta_1 p \tag{12}$$

qu'il faut substituer dans (5) et (10).

Dans le cas du mouvement établi, en excluant des équations obtenues le rapport $\nabla^2 p : \Delta_1 p$ on obtiendra une équation différentielle

$$\left(\frac{\varphi'}{\varphi} - \frac{f'}{f}\right) p \frac{d\sigma}{dp} + \frac{mc\sigma}{q_0} p + \varepsilon\beta cm \left[\frac{\sigma f(\sigma)}{\varphi(\sigma)}\right] + 1 = 0 \tag{13}$$

où l'on désigne

$$\varepsilon = \mu_2/\mu_1 \tag{14}$$

De cette équation on peut définir $\sigma(p)$ ou bien $p(\sigma)$. Dans le cas du mouvement non établi, nous pouvons, dans la première approximation, éliminer les membres contenant $\Delta_1 p$ comme étant de second ordre. En excluant des équations reçues le rapport $\nabla^2 p : \frac{\partial p}{\partial t}$ on obtient une équation différentielle

$$\left(\varepsilon + \frac{\varphi}{f}\right) p \frac{d\sigma}{dp} + \frac{c}{q_0}\left[\frac{\sigma\varphi(\sigma)}{f(\sigma)}\right] p + \varepsilon(1 - \beta c)\sigma - \varepsilon = 0 \tag{15}$$

servant à la détermination de $\sigma(p)$ et de $p(\sigma)$.

Les équations (13) et (15) peuvent être réduites à la forme

$$p \frac{d\sigma}{dp} = a_1(\sigma) + a_2(\sigma)\, p \tag{16}$$

où $a_1(\sigma)$ et $a_2(\sigma)$ sont des fonctions connues de la saturation σ.

Étant donné que dans les conditions réelles la pression p varie dans un intervalle étroit, on peut considérer approximativement que dans la partie droite de l'expression (16) p soit égale à sa valeur moyenne dans cet intervalle:

$$p = \bar{p} \tag{17}$$

On trouve alors la solution approximative de (16) sous la forme

$$p = \text{const}\, e^{\theta(\sigma)} \tag{18}$$

où

$$\theta(\sigma) = \int \frac{d\sigma}{a_1 + \bar{p} a_2}. \tag{19}$$

Manuscrit reçu
le 30.VIII.1945.

LITTÉRATURE CITÉE

[1] Л. С. Лейбензон, Нефтепромысловая механика, ч. II, гл. XI, М., 1934; idem, Bull. Acad. Sci. URSS, sér. géogr. et géophys. (1941). [2] M. N. Muskat and M. M. Meres, Physics, 7, September (1936). [3] R. D. Wyckoff and H. G. Botset, Physics. 7, September (1936). [4] L. S. Leibenson, Bull. Acad. Sci. URSS, sér. géogr. et géophys. (1945).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

PHYSIQUE MATHÉMATIQUE

RÉSOLUTION DU PROBLÈME DE REFROIDISSEMENT D'UN SEMI-ESPACE REMPLI D'UNE MATIÈRE DONT LES PROPRIÉTÉS SONT DÉPENDANTES DE LA TEMPÉRATURE

Par G. P. IVANZOV

(Présenté par S. L. Soboleff, de l'Académie, le 1. IX. 1944)

L'équation différentielle de Fourier pour le cas où la conductibilité λ et la chaleur spécifique c d'un matériel sont fonctions de la température, a été déduite en premier par Schwarz (1). On la retrouve ensuite dans les travaux de Helweg et Schmidt (2) et de Eigenson (3).

Si l'on désigne par c la chaleur spécifique de la substance, on a pour un cas à une dimension

$$\frac{\partial t}{\partial \tau} = a \frac{\partial^2 t}{\partial x^2} + \frac{1}{c\gamma} \frac{d\lambda}{dt} \left(\frac{\partial t}{\partial x} \right)^2 \tag{1}$$

Montrons qu'on peut trouver une solution de l'équation (1) sous la forme

$$t = F\left(\frac{x}{\beta \sqrt{\tau}} \right) = F(u) \tag{2}$$

Calculons t'_τ, t'_x et t''_x de l'expression (2) et substituons les valeurs trouvées dans l'équation (1). Nous obtenons une équation ordinaire non linéaire de second degré, qui peut être résolue en chaque cas concret par approximation

$$\frac{d^2 t}{du^2} + \frac{\beta^2}{a} u \frac{dt}{du} + \frac{1}{\lambda} \frac{d\lambda}{dt} \left(\frac{dt}{du} \right)^2 = 0 \tag{3}$$

En résolvant cette équation, il faut tenir compte que son dernier membre est généralement petit par comparaison aux deux premiers.

Aussi est-il rationnel de partir de la solution connue de l'équation (3), obtenue pour les valeurs constantes moyennes de a et de λ dans l'intervalle donné des températures

$$t = t_0 G\left(\frac{x}{\sqrt{4a_{\text{moyenne}} \tau}} \right) = t_0 G\left(\frac{\beta}{\sqrt{4a_{\text{moyenne}}}} u \right) \tag{4}$$

où G est une fonction de Kramp.

On peut résoudre le même problème d'une autre manière. Soient λ et c ainsi que t des fonctions de $u = x/\beta \sqrt{\tau}$.

Si λ et c sont fonctions de x et de τ, l'équation différentielle prend la forme

$$\frac{\partial t}{\partial \tau} = a \frac{\partial^2 t}{\partial x^2} + \frac{1}{c\gamma} \frac{\partial \lambda}{\partial x} \frac{\partial t}{\partial x} \tag{5}$$

Substituons dans cette équation les valeurs des dérivées. Nous obtenons une équation linéaire homogène ordinaire de second degré qui se résoud en quadratures

$$\frac{d^2t}{du^2} + \omega(u)\frac{dt}{du} = 0, \tag{6}$$

où

$$\omega(u) = \frac{\beta^2 u}{2a} + \frac{1}{\lambda}\frac{d\lambda}{du} \tag{7}$$

Ayant trouvé la solution approximée de (4) pour la valeur moyenne de a, substituons-la dans les expressions de λ et de c en fonction de t, pour obtenir leur expression en fonction de u.

Substituons cette dernière expression dans (7) et intégrons l'équation (6). On trouve ainsi la seconde approximation, *etc.*

Il est désirable de représenter $\omega(u)$ comme une série facilitant l'intégration de (6).

L'approximation successive pour les relations ordinaires de λ et c avec la température sera convergente.

Manuscrit reçu
le 1.IX.1944.

LITTÉRATURE CITÉE

[1] C. Schwarz, Z. f. techn. Physik, **9**, 457; **10**, 554 (1925). [2] E. Schmidt et F. Helweg, Forsch. a. d. Gebiete d. Ing., **4**, No. 5, 238 (1933). [3] L. S. Eigenson, Bull. Acad. Sci. URSS, sér. technique, No. 5, 85 (1941).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

PHYSICS

DIELECTRIC CONSTANT OF BARIUM TITANATE AS A FUNCTION OF STRENGTH OF AN ALTERNATING FIELD

By B. M. WUL, Corresponding Member of the Academy, and I. M. GOLDMAN

It has been established by the authors that the capacity of a barium titanate condenser, measured at a frequency of 50 cycles, increases severalfold with an increase in the field strength from small values to breakdown ones (1).

Since our first samples of barium titanate were porous, it looked like the increase in capacity could be explained by the ionization of the gas in the pores. In a medium with a dielectric constant $\varepsilon > 1\,000$, ionization of the air in the pores with an increase in voltage could have served as a sufficient reason for the abrupt increase in the value of the capacity. To check this hypothesis we prepared some samples of barium titanate at a higher annealing temperature. These samples had a dielectric constant $\varepsilon \cong 1\,800$ at room temperature under high frequency, and their porosity was extremely small. Measurements made with these samples showed that the capacity of these condensers is likewise dependent upon the field strength. If the residual, though negligibly small porosity had a noticeable effect in this case, irradiation of the sample by a powerful source of γ-rays should affect the dependence of the capacity upon the applied voltage. However, irradiation of the sample with a 100 mg radium preparation at a distance of 15 cm did not have any effect.

In order to confirm that the change in capacity with an increase in voltage is not due to the ionization of the residual pores but rather to a change in the dielectric constant of the barium titanate itself, we measured the dependence of the capacity upon the voltage at room temperature and at the temperature of liquid oxygen; the dielectric constant of barium titanate is seven times as great in the first case as in the second.

Hence, if the residual porosity played a noticeable rôle, the relative increase in capacity would be appreciably higher at room temperature than at the temperature of liquid oxygen. Measurements showed that upon passing from low voltages to high, where the capacity reaches a maximum, the increase is 4—5 times at room temperature and 25 times at the temperature of liquid oxygen. Thus, it may be regarded as proved that in an alternating field of 50 cycles the dielectric constant of the barium titanate itself changes with a change in the field strength. The dependence of ε upon the field strength for the samples we studied is given in Fig. 1. The measurements were carried out on a Shering bridge, an amplifier being used at small voltages.

As can be seen from the data given in Fig. 1, the initial value of the dielectric constant at room temperature is $\varepsilon \cong 1\,800$, at a field strength of 7 kV/cm the dielectric constant reaches a maximum and exceeds 8 400; with a further increase in voltage ε shows a tendency to decrease, which is not observable for long owing to breakdown which sets in at about 15.5 kV/cm.

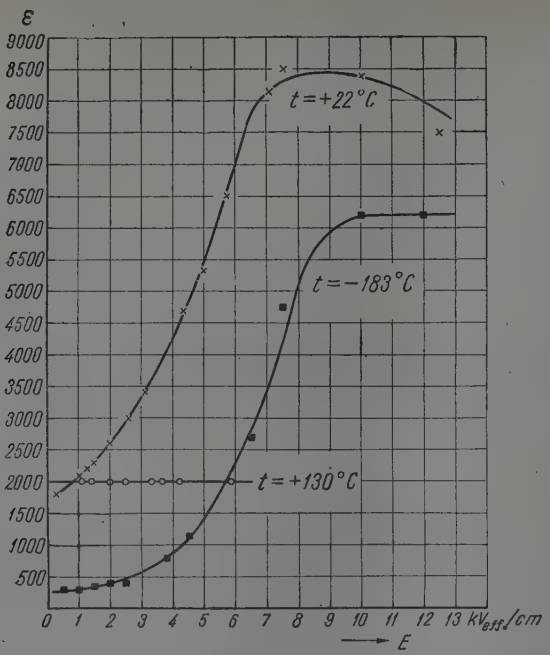

ε
9000
8500
8000
7500
7000
6500
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
t = +22 °C
t = −183 °C
t = +130 °C
0 1 2 3 4 5 6 7 8 9 10 11 12 13 kV/cm
E

Fig. 1

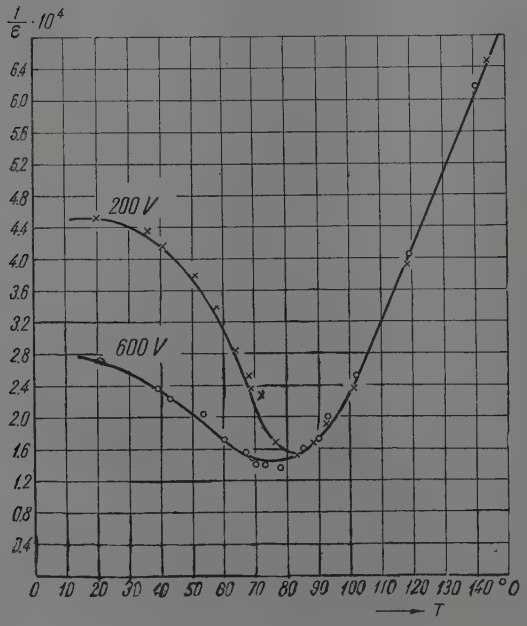

1/ε · 10⁴
6.4
6.0
5.6
5.2
4.8
4.4
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0.4
200 V
600 V
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 °C
T

Fig. 2

In liquid oxygen ε has an initial value about 250, risen to 6200 at a field strength of about 10 kV/cm, and remains at this value until 15 kV/cm are reached. Since the capacity depends upon the voltage, the values of ε cited are effective values computed from the values of the capacity measured on a bridge having for indicator a vibration galvanometer tuned in resonance to a frequency of 50 cycles.

As we have shown, the dielectric constant of barium titanate at a high frequency markedly depends upon the temperature and has a very distinct maximum ([1]). The temperature corresponding to the maximum depends upon the annealing temperature and the purity of the initial materials. At 50 cycles, as well as at a frequency of $1.5 \cdot 10^6$ cycles, the temperature corresponding to the maximum for the given series of samples was 80°C. To elucidate the nature of the temperature maximum N. K. Voskresenskaya and V. J. Anosov of the Institute of General and Inorganic Chemistry of the Academy of Sciences of the USSR made some measurements of the specific heat of barium titanate at our request, and found an abrupt variation of the specific heat at a temperature of about 125°C.

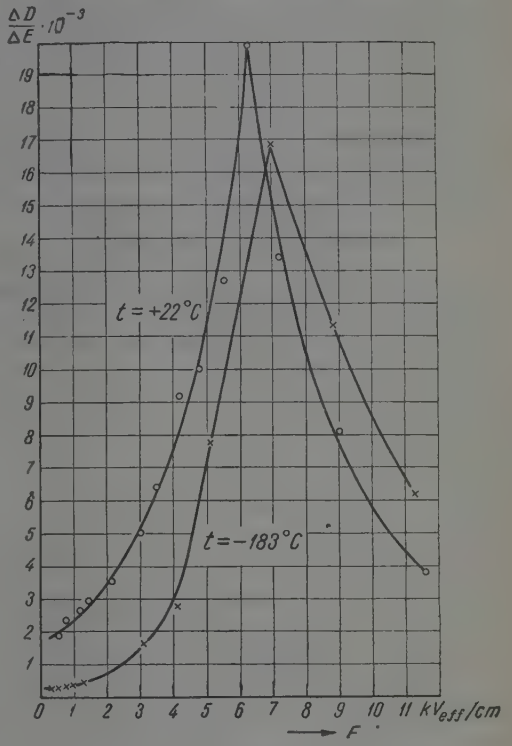


Fig. 3

With the same purpose of elucidating the nature of the temperature maximum of ε we measured the dependence of ε upon the temperature at different voltages. Data obtained at 200 and 600 V with a sample 1.8 mm thick are given in Fig. 2. As can be seen from these data, at a temperature exceeding Θ—the temperature corresponding to the maximum ε, the dielectric constant of barium titanate depends no longer upon the applied voltage. Additional measurements at 135°C and voltages ranging from 65 to 1150 V fully confirmed this. The results of these measurements are given in Fig. 1.

Thus, it may be considered as proved that in an alternating field of 50 cycles the dependence of the dielectric constant of barium titanate upon the field strength changes at $T = \Theta$. This gives grounds for the assumption that the structure of barium titanate changes when $T < \Theta$. V. L. Ginsburg in an unpublished work on the dielectric properties of new Rochelle salt compounds ([2]) and barium titanate, advanced the assumption that when $T < \Theta$ barium titanate is pyroelectric, and that when $T = \Theta$ a transformation from the pyroelectric into the non-pyroelectric modification takes place. Basing himself on the work of L. D. Landau on phase transformations of the second kind, V. L. Ginsburg arrives at certain quantitative relations which cannot be checked until additional experimental data are available.

When the temperature $T > \Theta$, ε did not depend upon the field strength and the values we have computed from experimental data correspond to the usual value $\varepsilon = D/E$, where D is flux density, E is the field strength.

As can be seen from Fig. 2, when $T > \Theta$, $\frac{1}{\varepsilon} = a + b(T - \Theta)$, *i. e.* a relation

analogous to the Curie-Weiss law holds. When $T<\Theta$, ε depends upon the field strength, and in this case the value of the dielectric constant determined as $dD/dE=\varepsilon'$ will be different from the values we have computed and given in Fig. 1. Having measured the capacity as a function of the voltage, we could compute the values of D and take the $\frac{D_i-D_{i-1}}{E_i-E_{i-1}}$ instead of the values ε' corresponding to the field strength $\frac{E_i+E_{i-1}}{2}$. These data for the temperatures $+22°C$ and $-183°C$ are given in Fig. 3. They show that ε' reaches a maximum at a field strength of 6—7 kV/cm and that the position and value of the maximum on the curve $\varepsilon'=\varphi(E)$ varies but slightly with the temperature.

The samples were prepared and the measurements carried out with the assistance of R. J. Razbash and N. L. Strogonova.

The authors are indebted to N. K. Voskresenskaya and V. J. Anosov for the measurements of the specific heat. Thanks are also due to N. V. Belov for the X-ray studies.

The P. N. Lebedev Institute of Physics.
Academy of Sciences of the USSR.

Received
27.VII.1945.

REFERENCES

[1] B. M. Wul and I. M. Goldman, C. R. Acad. Sci. URSS, XLVI, 154 (1945).
[2] G. Busch, Helv. Phys. Acta, **11**, 269 (1938).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

PHYSICS

ENERGY DISTRIBUTION IN THE SPECTRUM OF A BLACK RADIATOR AND WIEN'S DISPLACEMENT LAW

By S. ORLOV, Corresponding Member of the Academy

The quantity of energy emitted in a frequency interval between ν_1 and ν_2 from a square centimetre of the surface of a black radiator at temperature T in a second is expressed by a well-known formula of Planck

$$E = \int_{\nu_1}^{\nu_2} \frac{2\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}\, d\nu \qquad (1)$$

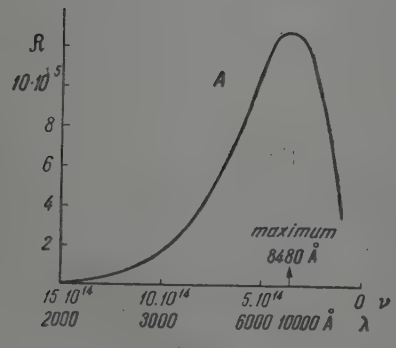


Fig. 1

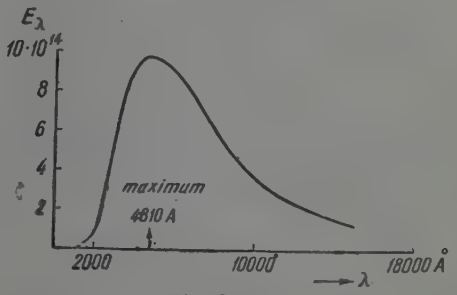


Fig. 2

The frequency ν can be replaced by the wave-length λ with which it is connected by the relation $\nu = c/\lambda$. We shall then have

$$E = \int_{\lambda_1}^{\lambda_2} \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}\, d\lambda \qquad (2)$$

The above two integrals are identical and yield equal numerical values, provided, of course, that the ranges of integration have been brought into correspondence. The integrands were described by Planck, respectively, as radiation intensity with respect to frequencies $\mathfrak{R}_\nu = \frac{2\pi h\nu^3}{c^2}(e^{h\nu/kT} - 1)^{-1}$, and radiation intensity with respect to wave-lengths $E_\lambda = \frac{2\pi hc^2}{\lambda^5}(e^{hc/\lambda kT} - 1)^{-1}$.

In § 19 of his book Planck (1) points out that the radiation intensity E_λ is not obtained by mere substitution in the expression for $\mathfrak{R}_\nu$ of the value of ν with that of λ by means of formula $\nu = c/\lambda$. Because we have not $\mathfrak{R}_\nu = E_\lambda$,

but rather $\mathfrak{K}_\nu d\nu = E_\lambda d\lambda$, if $d\nu$ and $d\lambda$ refer to the same interval of the spectrum. In § 89 of the same book he remarks that the maximum of $\mathfrak{K}$ does not coincide in the spectrum with the maximum of E_λ. It may be added to this that $\mathfrak{K}_\nu$ and E_λ have different dimensions.

The dimensions of the integrals themselves correspond to the power of radiations from a cm^2 $[M^1T^{-3}]$. In Figs. 1 and 2 these integrals are represented graphically; the curves are for 6000°K (the effective temperature of the Sun). The dimension of the integrand $\mathfrak{K}_\nu$ corresponds to the energy emitted by a cm^2 $[M^1T^{-2}]$ (the dimension of h is $M^1L^{-1}T^{-2}$, the dimension of ν is T^{-1}). Function E_λ has the dimension $M^1L^{-1}T^{-3}$, so instead of energy it represents the power of radiation from a square centimetre, referred to unit length. Since $\mathfrak{K}_\nu$ (the ordinate of the curve A) has the dimension of the energy, curve A, but not B, as is generally assumed, may be taken to represent the energy distribution over frequencies in the spectrum of a black radiator.

The two curves A and B are both legitimate, they both bring out the distribution in the spectrum of the black body of quantities proportional to the power of the emitted energy, with the only difference that in the case of curve A this power is computed all over the spectrum for a constant interval $\Delta\nu$, whereas in the case of curve B the interval is $\lambda\Delta$. Wien's displacement law, if written in usual terms ($\lambda_m = 0.289/T$) refers to curve B. This curve is all too often considered to be unique because one is apt to forget the existence of another curve, *viz.* curve A which is no less good, but for which Wien's law has a different constant ($\lambda_m = 0.509/T$).

It has been the general custom upon studying by experiment the energy distribution in the spectrum of a black radiator to refer the quantity of energy absorbed by the receiver to some arbitrarily chosen constant interval of the spectrum $\Delta\lambda$ (2). It is therefore the curve B that has received experimental confirmation, and this curve is regarded as the only one representing the energy distribution in the spectrum of a black radiator. For the Sun, for instance, whose effective temperature is of the order of 6000 K, the maximum power of radiations is considered to lie at the wave 4810 Å ($\lambda_m = 0.289/T$). But we must not forget that this result is obtained by computing the power for a constant spectral interval $\Delta\lambda$. If, on the other hand, $\Delta\nu$ is taken for the constant interval, then the radiation maximum falls on the wave 8480 Å ($\lambda_m = 0.509/T$). Both of these numbers are valid, and we must choose between them according to the case in hand.

The usual formulation leads to misunderstandings, as the following example will show.

The radiation pressure on a molecule is a function of the frequency of the characteristic radiation which the molecule absorbs. The repelling acceleration of the Sun is defined in units of Newtonian acceleration by the formula (3)

$$1+\mu = \frac{2\pi e^2 h\nu^3 f_{12}}{m_e M g_\odot c^4} \cdot \frac{1}{e^{h\nu/kT}-1}$$

where e is the charge of an electron; m_e, its mass; f_{12} is the force of the oscillator which on resonance transitions is always near to one half; M is the mass of the molecule; $g_\odot$ is Newtonian acceleration at the Sun's surface. This formula can be re-written as follows

$$1+\mu = \frac{\pi e^2 f_{12}}{m_e M g_\odot c^2}\,\mathfrak{K}_\nu$$

The coefficient before $\mathfrak{K}_\nu$ being constant for a given molecule, there should be coincidence between the maxima of the functions $1+\mu$ and $\mathfrak{K}_\nu$.

By means of this formula Wurm (4) has computed the radiation pressure on a CO^+ molecule taking for the characteristic radiation the frequency ν which corresponds to $\lambda = 4900$ Å. From the fact that the maximum of energy in the

solar spectrum, according to the generally adopted expression of Wien's distribution law, lies very close to $\lambda = 4900$ Å ($\lambda_m = 4810$Å), Wurm was led to believe that the value $1+\mu = 54$ obtained by him was the highest possible

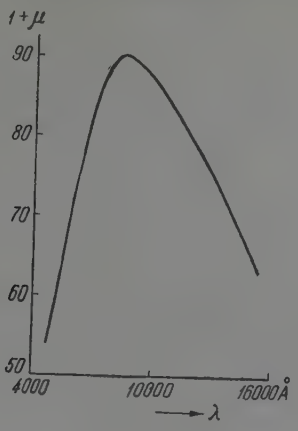


Fig. 3

one. As a matter of fact, the maximum value is $1+\mu = 90$, and it corresponds, as might be expected, to the wave $\lambda = 8480$ Å where the maximum of the function $\mathfrak{K}_\nu$ lies (Fig. 3).

In order to avoid such errors we must take for the constant in Wien's distribution law the value 0.509 if the effect under consideration is proportional to $\mathfrak{K}_\nu$, and the generally adopted value 0.289 if the effect is proportional to E_λ.

Received
7.VIII.1945.

REFERENCES

[1] M Planck, Vorlesung über die Theorie des Wärmestrahlen, Leipzig, 1923. [2] J. Wilsing, Publ. Ap. Obs. Potsdam, No. 72, 5. [3] В. А. Амбарцумиан, Теоретическая астрофизика, 1939, стр. 212. [4] K. Wurm, Z. f. Aph., **10**, 288 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 8

HYDROLOGY

FORMATION OF STORM FLOODS

By M. A. VELIKANOV, Corresponding Member of the Academy

Under the climatic conditions prevailing in the greater part of this country a rainstorm can cause flooding only on very small watersheds. This gives ground for two assumptions which in the case of a small drainage area are quite natural. Let us formulate them as follows:

a) The soil characteristics are supposed to be invariable throughout the area of the basin both with respect to roughness (which influences the velocity of running) and with respect to infiltration.

b) The whole basin is supposed to be overhung by the storm cloud and the intensity of the rain within this area is everywhere the same, being considered to be only a function of time.

The first of these assumptions hardly needs any comment, but concerning the second it may be remarked that even in the case of a denser rain gauge net than is used at present the possibility of measuring the intensity at several points of a small drainage area seems rather doubtful. If so, we shall certainly gain very little from the practical point of view if we refine our theory by dropping the second assumption.

The entire process of flood formation for a particular watershed and for a certain intensity given as a function of time can depend solely on two factors: 1) the velocity and rate of flow of the precipitated water, and 2) the rate of loss by percolation into the soil. The relief factor can, as has been shown by the author in an earlier paper [1], be taken into account by means of a special characteristic, the time of running. This quantity is represented by the integral

$$\tau = \int_{(S)} \frac{dx}{u} \tag{1}$$

taken along the line of flow. Obviously, it should itself depend upon the time (the usual solar time), since the variations in the intensity affect the hydraulic condition of the flow, hence, the value of the integrand.

We shall consider the discharge of water in a definite section of the stream at a time t. It will apparently include the particles flowing off from the different elements of the entire area of the watershed, and for every particle the different moments t' at which the flowing starts must satisfy the equality

$$t' + \tau = t\,(\text{const}) \tag{2}$$

If for every instant t we imagine a curve bringing out the function $\omega(\tau)$ where ω is the area bounded by the $\tau = \text{const}$ (isotime curve), the watershed line and the

closing section, then we shall have a function of two variables $\omega(\tau,t)$ which can now be considered arbitrarily as independent variables.

Let us now turn to the analysis of the second factor in the formation of a flood, *viz.* the losses by percolation into the ground.

On the question of its estimation two diametrically opposite views have been expressed in literature. One method, put forward by Protodiakonov (²), is to subtract the layer of losses from the precipitation layer, or, in other words, to have the difference $(h—p)$ in the computation formula, where h is the intensity of the rain, and p is the rate of percolation. According to the second view, advocated by Sribny(³), it is always a definite portion of the precipitated water that percolates into the ground, therefore the computation formula must contain the product of the rain intensity and a coefficient varying according to the kind of the ground and the microrelief of the surface. It may easily be realized that this different treatment will lead to widely diverging results in the calculation. For large rain intensities the first method will certainly give an overestimated value of infiltration, and for small intensities the second method errs in this direction. Neither reflects the physical nature of the phenomenon. As a matter of fact, every particle of water fallen to the surface loses in mass by percolation in proportion to the time during which it flows down. This simple consideration has led the author to the following method whereby the infiltration is taken into account and which is based on a new principle.

Let us consider an elementary volume v composed of water particles that fell at time t on an elementary area δf and are moving along the line of flow (S).

Taking the velocity of infiltration, by analogy with the Darcy law, to vary directly as the height of the flow layer H, we obtain for the rate of infiltration referred to a time element δt the quantity

$$\delta v = -\delta f H k \delta t = -kv\delta t$$

where k is an empirical coefficient which, according to assumption a), is constant for a given watershed.

Thus, we have the relation $dv/dt = -kv$, which upon integration gives the equation

$$v = v_0 e^{-kt} \tag{3}$$

This is the formula by which the authors suggest to estimate the percolation losses.

Let us now consider the volume of water that passed through the closing section during the interval from t to $t+\delta t$

$$\delta V = Q\delta t$$

where Q is the discharge per second. We imagine this volume to be divided up into a very great number N of very small volumes

$$Q\delta t = \sum_N v_j$$

To every one of these volumes we mentally assign a number and refer it to a certain place and time when it began to move down the slope, that is to a certain element of the drainage area δf_j fixed by the coordinates of its centre x_j, y_j, and to a certain time interval $(t'_j, t'_j+\delta t'_j)$.

The initial value of every individual volume when it starts flowing down can be taken to be equal to

$$\delta f_j h_i \delta t'_j$$

and its final value, after subtracting all loss sustained by percolation in the running during the time interval τ_j, will be, accordingly

$$v_j = \delta f_j \delta t' h_{t'} e^{-k\tau_j}$$

The sum of these small volumes will give us the initial volume δV. It is a convenience to distribute this sum into partial sums by uniting the terms with equal times of running. A partial sum thus taken will correspond to the area comprised between the isotime curves τ, $\tau + \delta\tau$, and will be equal to

$$\frac{\partial \omega}{\partial \tau} h_{t-\tau} e^{-k\tau} \delta\tau \delta t$$

The sum of these partial sums is represented by an integral which after reduction by the parameter δt gives

$$Q_t = \int_0^{\tau} \frac{\partial \omega}{\partial \tau} h_{t-\tau} e^{-k\tau} d\tau \qquad (4)$$

It will be obvious that the upper limit of integration corresponds to the greatest time of running among those characterizing the elementary volumes entering into the total volume δV. Clearly, this time of running, the greatest for the discharge Q_t, is equal to the interval elapsed from the start of raining to the instant t. And if it is agreed upon, for the sake of simplicity, to measure the solar time from the start of the rainstorm, then we will have $\tau = t$ for the upper limit of integration in (4).

What is described by the equation (4) is the entire process of formation of a flood discharge in a section closing a small watershed with a homogeneous soil cover, completely overcast by a storm cloud and beaten by a rain whose intensity is invariable from place to place and is considered to be only a function of time.

In order to give this equation a form suitable for practical calculations we must have the relation

$$\omega = \omega(\tau, t)$$

Let us consider the question as to the probable form of this relation and represent it as a family of curves $\omega(\tau)$ with parameter t. Some studies made under the author's guidance give ground to believe that for small areas the curves $\omega(\tau)$ are represented approximately by the following kind of parabola

$$\omega = m\tau^n$$

The exponent n seems to vary little as we pass from one intensity to another, and only brings out the increase or decrease of the areas comprised between the adjoining isotime curves as we move away from the closing section. On the other hand, the parameter m clearly increases with the rain intensity, which, indeed, it might have been expected to do. This relation, too, can be represented by the above type oi parabola

$$m = m_0 h^s$$

Here we will introduce an assumption against which there appears to be no objection. We shall suppose that the parameter m has its dependence on time t fully described by the above mentioned relation between itself and the variable with time intensity of the rain. Thereby the strict equality (4) transforms into an approximate equality

$$Q = n m_0 \int_0^{\tau=t} \tau^{n-1} h_{t-\tau}^{1+s} e^{-k\tau} d\tau \qquad (5)$$

from which the Q_t-curve can be plotted by the methods of numerical integration, if the drainage area in question is given by its morphometrical parameters n, m_0, s, its soil parameter k and, of course, if the rain intensity curve $h(t)$ is known.

In a vast majority of practical cases we have no knowledge of the variation of the rainstorm and can instead avail ourselves of two parameters only: the average intensity h, and the total duration T. In other words, $h=0$ for $t<0$ and $t>T$, while inside of the interval $(0-T)$ we have $h=\text{const}$. Then the integral (5) has to be divided up into two integrals

$$\text{a)}\quad t \leqslant T \qquad Q = nm_0 h^{1+s} \int_0^{\tau=t} \tau^{n-1} e^{-k\tau} d\tau$$

$$\text{b)}\quad t \geqslant T \qquad Q = nm_0 h^{1+s} \int_{t-T}^{t} \tau^{n-1} e^{-k\tau} d\tau$$

Both of them by substituting $k\tau = x$ can be represented as not complete gamma-functions, whereby all computations of the Q_t-curve are greatly simplified.

Received
11.VII.1945.

REFERENCES

[1] М. А. Великанов, Журн. геофизики, № 1—2 (1931). [2] М. М. Протодьяконов, Сборн. под ред. Р. Д. Дубелира «Максимальный сток с малых бассейнов», стр. 11, 1941. [3] М. Ф. Срибный, там же, стр. 54.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

GEOPHYSICS

ILLUMINATION OF THE EARTH SURFACE BY ATMOSPHERE SCATTERED RADIATION, ULTRA-VIOLET, GREEN AND RED, AS AFFECTED BY THE STATE OF THE SKY

By N. N. KALITIN

(Communicated by A. A. Grigoriev, Member of the Academy, 15. XII. 1944)

It has been shown by the author in an earlier paper (1) that the ratio between the illumination due to the total flux of radiation and that due to its ultra-violet portion is liable to considerable variation, according to the cloudiness of the sky.

It is certainly interesting and important from the practical point of view to study this relationship for different regions of the spectrum. A work of that kind was undertaken by the author in the summer period of 1944 at the same place where his former experiments (of 1943) had been carried out. This time the illumination of the Earth surface by the scattered radiation of the atmosphere under varying conditions of cloudiness was measured separately in three regions of the spectrum: the ultra-violet, the green and the red.

For measuring the illumination in these regions three photoelectric (selenium) photometers were built, all of the same type (photocells as designed by K. G. Trofimov), by means of which illumination from the whole sky could be measured. With the help of filtres the following portions of the spectrum were isolated: ultra-violet 3000—4000 Å, green 5200—6500 Å, and red 6000—7000 Å. In all observations the readings of the galvanometers were taken every 30 seconds and continuously during 20 minutes. At a few minutes intervals the cloudiness was estimated both quantitatively and with reference to form, and records were taken of its distribution over the sky. The cloudiness records followed each other at such intervals as to secure for the whole period of observations a detailed material on the state of the sky as regards cloudiness.

In this way data were collected to study the relationship between the state of the sky and the variation of the illumination in the three chosen regions of the spectrum. In all, 72 series of observations were made. As the illumination was measured in relative units, the following procedure was adopted in order to obtain comparable results. In every series of observations the first readings of the galvanometers were put at hundred, and the subsequent readings were expressed in per cent relative to the first.

From the outset it was found that in a vast majority of cases the variations are most conspicuous in the red part of the spectrum, they are less so in the green part and least of all in the ultra-violet. But sometimes the distribution was different, and the average ratio for all cases is as follows: red, 40 per cent; green, 32 per cent; ultra-violet, 28 per cent.

Under a cloudless sky the illumination varies but little in magnitude. In the green region of the spectrum the variations are more marked than in the others. Yet, for this state of the sky only a few observations are available. Under a sky covered by low unbroken clouds, such as St, Sc, Ns, the variations, if any at all, are small and equal for all three regions of the spectrum. As for the changes in illumination under conditions of variable cloudiness, they differ within a very wide range according to the amount of clouds, their form and distribution over the sky.

The influence of cloudiness on the variation of illumination in the investigated regions of the spectrum is brought out by Fig. 1. In this case the conditions

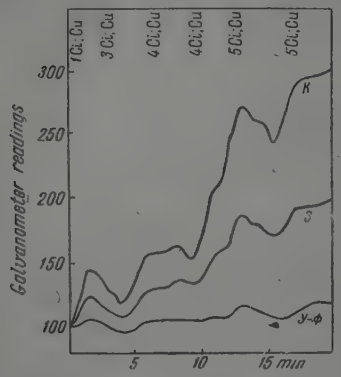


Fig. 1. Illumination under Cu clouds

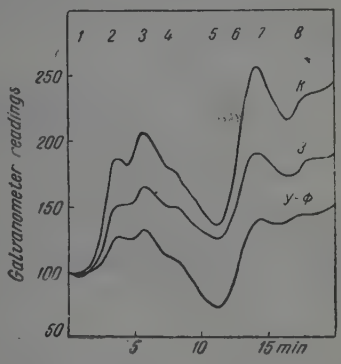


Fig. 2. Illumination under Ci, Cu clouds

of observation were as follows. The elevation of the Sun, 55°. At the outset the sky was practically cloudless. Cloudiness, 1 Ci; Cu near the horizon, which of course had little effect on the illumination of the horizontal surface. Some time later clouds of the Cu form began rapidly to appear, mainly in the region of the zenith. In 20 minutes, *i. e.* by the end of observations, they attained a quantity estimated by 5 points (a small amount of Ci all the time remained near the horizon). Fig. 1 shows that in the ultra-violet the illumination was practically invariable, while in the green it had grown up to 90 per cent, and in the red, up to 200 per cent.

From the run of the curves it may be inferred that if the observations had been continued under increasing cloudiness, the increase of illumination in the green and red rays would have been found to be greater still. The ratio between the variation of illumination in the three spectrum regions investigated can vary greatly, according to the state of cloudiness at the initial moment and its variation afterwards in the course of observations. But, as has been mentioned above, the strongest changes are observed in the red region and the most active part of the sky is the zenith.

If the wind at the altitude of the clouds has a small velocity, the illumination varies slowly. In the presence of a strong wind the variation may be very rapid. Even without a wind the illumination can vary greatly in different parts of the spectrum, provided that dynamical processes are at work controlling the appearance and disappearance of clouds; of cumuli, for instance.

With clouds of complex forms the variation of the illumination can be very considerable, as Figs. 2 and 3 show. Fig. 2 represents the results of measurements made on June 3, 1944. The elevation of the Sun was 53°. Clouds of the Cu-form were moving very rapidly. The figures indicate the instants for which the cloudiness is given in Table 1, and the lettres indicate the regions of the spectrum (the same notation is used in the other drawings as well).

Table 1

1 — 7 Ci; Cu (3 Cu); no clouds in the zenith
2 — 7 Ci; Cu (4 Cu); Cu in the zenith
3 — 8 Ci; Cu (5 Cu); bright Cu in the zenith
4 — dark Cu are approaching the zenith
5 — 9 Ci; Cu (8 Cu); dark Cu in the zenith
6 — 9 Ci; Cu (7 Cu); dark Cu are moving away from the zenith
7 — 8 Ci; Cu (7 Cu); patches of blue sky in the zenith
8 — 8 Ci; Cu (7 Cu);

Fig. 3 brings out the variations in illumination observed at a Sun-height of 54° on July 15, 1944. The conditions in the sky as regards cloudiness were very complex. All over there was a dense uniform layer of Cs. Lower down some Ac were present, and beneath them were 3—6 Cb. These latter clouds were dark and moving with great speed.

Table 2

1 — 10 Cs; Ac; Cb (4 Cb)
2 — 10 Cs; Ac; Cb (6 Cb); dark Cb in the zenith
3 — 10 Cs; Ac; Cb (4 Cb); dark Cb moving away from the zenith
4 — 10 Cs; Cb (4 Cb)
5 — 10 Cs^2; Cb (3 Cb); dense Cs in the zenith
6 — 10 Cs^2; Cb (3 Cb)
7 — 10 Cs^2; Cb (3 Cb)

In this case the illumination varied differently in different regions of the spectrum. Supposing that when the first record was taken the illumination was equal in all parts of the spectrum, the decrease observed five minutes later, when dark Cb occupied the region of the zenith, was 46 per cent for the ultra-violet, 43 per cent for the red, and 30 per cent for the green. Ten minutes later, when the dark Cb clouds had left the zenith, the illumination was greatest in the ultra-violet rays, less in green, and still less in red. The last records showed a different distribution again: the illumination was a maximum in the ultra-violet region, a minimum in the green, and the red came between.

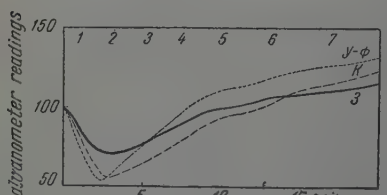


Fig. 3. Illumination under Cs, Ac, Cb clouds

What has been brought into evidence by this preliminary investigation is that the illumination of the Earth surface by the scattered light of the atmosphere may vary greatly in different parts of the spectrum according to cloudiness, and that this variation has often a rather peculiar character. Further inquiry into the matter is necessary for the elaboration of practical methods whereby proper correction of aerial survey observations will be made possible. The results will also be of interest for the workers engaged in the study of the photosynthetic activity of plants under natural conditions.

Received
15.XII.1944.

REFERENCES

[1] N. N. Kalitin, C. R. Acad. Sci. URSS, XXXIX, № 2 (1943).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

CHEMISTRY

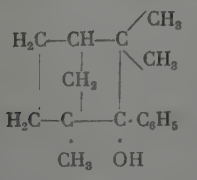
ON PHENYL-FENCHOCYCLENE AND SOME OF ITS TRANSFORMATIONS

By S. S. NAMETKIN, Member of the Academy, and S. I. OBTEMPERANSKAYA

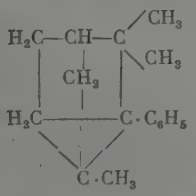
As early as 1909 Leroide and Haller using the reaction of phenyl-magnesium bromide with fenchone obtained the tertiary phenyl-fenchyl alcohol (I) and, upon its dehydration, a hydrocarbon $C_{16}H_{20}$ but the nature of the latter was not established (1).

It is possible from the theoretical point of view to regard dehydration as proceeding in two directions: either with a profound rearrangement of the camphene type, leading to one of the possible phenyl-fenchenes (or to their mixture), or without a rearrangement, when water is split off in position 1,3 with the formation of the tricyclic phenyl-cyclofenchene.

A special experiment demonstrated that the hydrocarbon formed when the tertiary phenyl-fenchyl alcohol (I) was heated with sodium bisulphate did not react with benzoyl hydroperoxide, according to N. A. Prilezhaev, and therefore contained no double bond. Thus, a tricyclic structure should be ascribed to this hydrocarbon, or more precisely, the structure of phenyl-cyclofenchene (II).



(I)



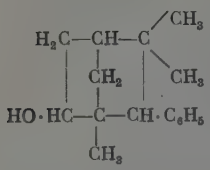
(II)

The properties both of the tertiary alcohol (I) and of the hydrocarbon (II) corresponded to the constants of compounds, described by Leroide and Haller (1).

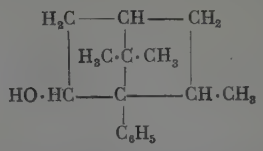
How would proceed the dehydration of phenyl-fenchocyclene? We set out to find an answer to this question expecting that here, just as in the case of hydration of several other tricyclic hydrocarbons, it would be possible to arrive at the synthesis of new phenylated homologues of the camphor group.

An attempt to hydrate phenyl-cyclofenchene according to Bertram and Wahlbaum (with acetic acid in the presence of small amounts of sulphuric acid) gave negative results. Under these conditions phenyl-fenchocyclene underwent no change. Then a stronger acid, trichloroacetic, was used in order to achieve esterification, and under these conditions the reaction did take place. No attempt was made to isolate the ester in a pure state, and the raw product obtained in the reaction of phenyl-cyclofenchene with trichloroacetic acid was hydrolysed with an alcoholic solution of alkali, and a secondary alcohol was thus obtained. From the

structure of phenyl-cyclofenchene (II) given above one comes to the conclusion that the most probable structural formula for this alcohol is either (III) or (IV).

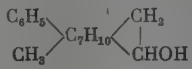


(III)



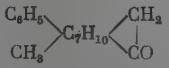
(IV)

We have no sufficient data as yet for a final choice between the two formulae. Accordingly, for expressing the structure of the new secondary alcohol obtainned by us, as well as of the products of its subsequent transformations, we shall use formulae that have not been developed to the full. Thus, for the product of hydration of phenyl-cyclofenchene we shall use formula (V).



(V)

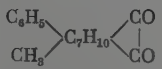
This secondary alcohol is a colourless crystalline substance, which after crystallization from light gasoline has a m. p. 125—125.5°C. When the solvent is evaporated slowly, the alcohol separates as large crystals, being a combination of a hexahedral prism and truncated pyramid



(VI)

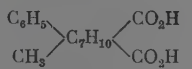
Oxidation of the secondary alcohol (V) with dichromic acid yielded the corresponding ketone (VI) which is a colourless crystalline substance, well soluble in all organic solvents. After purification by way of the semicarbazone the ketone had a m. p. 65—65.5°.

The semicarbazone of the ketone (VI) is formed easily under usual conditions. After two crystallizations from methanol the semicarbazone was obtained in needles, m. p. 193—194°.



(VII)

Oxidation of the ketone (VI) with selenium dioxide in an acetic acid solution, according to Asachina, Ishidate and Mumose (2), yielded the corresponding quinone (VII). When crystallized rapidly from glacial acetic acid, the quinone separated in fine plates coloured yellow, m. p. 144—145°. When crystallized slowly from the same solvent, the quinone formed flat hexahedral prisms.



(VIII)

When the quinone (VII) was treated with a 30 per cent hydrogen peroxide solution according to Holleman (3), the anhydride of a new dicarboxylic acid was obtained as a colourless finely crystalline substance, m. p. 165—176°. Upon a slow crystallization from acetone the anhydride separated in flat rhombic plates. This substance was dissolved in a potassium hydroxide solution, and the latter was acidified until a positive reaction for tropaeolin was reached. A crystalline acid (VII) separated that had m. p. 171—172° in a sealed tube.

All the substances described above have been obtained in an analytically pure state.

Conclusions. 1. The nature of the product of dehydration of tertiary phenyl-fenchyl alcohol has been established, and the product has been proved to be phenyl-cyclofenchene.

2. The hydration of phenyl-cyclofenchene has been achieved successfully by the action of trichloroacetic acid with subsequent hydrolysis of the ester.

3. Several representatives have been obtained of a new series of phenylated homologues of the camphor group: a secondary alcohol, a ketone, a quinone, a dicarboxylic acid. Their structure calls for more accurate definition.

The M. V. Lomonosov State University.
Moscow.

Received
14.VII.1945.

REFERENCES

[1] Leroide et Haller, C R., **148**, 1611 (1909). [2] Asachina, M. Ishidate, T. Mumose, Ber., **67**, 1432 (1934). [3] Holleman, Rec., **23**, 169 (1904).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

PHYSICAL CHEMISTRY

THE RATE OF CORROSION OF ALUMINIUM DEPENDING ON THE pH OF THE SOLUTION

By G. V. AKIMOV, Corresponding Member of the Academy, and A. I. GLUKHOVA

To find out the influence of external conditions upon the corrosion of a given metal is always a matter of considerable interest both from the theoretical and practical point of view. With regard to corrosion the behaviour of a metal is essentially characterized by the diagram expressing the dependence of the rate of corrosion on the pH of the solution. It has long been known that aluminium corrodes slowly in neutral solutions and is attacked very rapidly both in strongly alkaline and in strongly acid solutions. Yet a complete picture in the form of a K—pH diagram is lacking, and it is the object of the present paper to fill up the gap. In solving the problem it was important to take into consideration the influence of the anion of the solution as well. We know perfectly well that the corrosion behaviour of aluminium depends to a considerable degree upon the relationship existing between the ions of the solution and the protective film. Accordingly, differences in the penetrating power of the anions must be reflected in the position of the points on the K—pH diagram. In course of our experiments three series of measurements have been carried out in the presence of three different ions Cl'_2, SO''_4, and NO'_3. The solutions were prepared as follows.

A solution of the sodium salt of the corresponding anion (*i. e.* NaCl, Na_2SO_4 or $NaNO_3$) was acidified with HCl (or H_2SO_4, and HNO_3) or made alkaline with NaOH in order to obtain the necessary value for pH and at the same time to keep the solution 1 *n* in respect to the given anion. Corrosion was measured using the gravimetric method. The duration of the experiment in case of neutral, weakly acid or weakly alkaline solutions was 240 hrs.

In strongly acid and strongly alkaline solutions, where the process taking place was very intense, we had to confine ourselves to an interval of 24 hrs. Simultaneously with the study of the corrosion behaviour in a medium with different pH values we decided to measure the electrode potentials *E* of aluminium in the course of corrosion. For that reason our samples—plates $30 \times 15 \times 0.5$ mm—were fitted with a wire, protected by a glass tube and rubber wax *. As comparison electrode a calomel electrode was used. In the diagram all the values of potentials have been recalculated on the hydrogen scale. Every experiment was repeated at least twice, and the average was taken.

During the corrosion process a change in the pH of the solution took place, notwithstanding the fact that the volume of the latter compared to the surface of the samples was considerable (125 cm^3). A particularly great change in pH was observed after 240 hrs both with weakly acid solutions (up to 2 units of pH in the direction of decreasing acidity) and with weakly alkaline (up to 2 units in the direction of decreasing alkalinity). In the diagram the i n i t i a l values of pH

* The composition of aluminium was as follows: Fe, 0.26; Si, 0.20; Cu, 0.30; Al, the rest.

are reported, corresponding to the particular normal concentration of the acid or alkali.

The results of our measurements are expressed in Fig. 1. In this figure curves are compared showing the dependence of the rate of corrosion on the pH of the solution for two anions Cl' and SO_4'' (in semi-logarithmic coordinates), and cur-

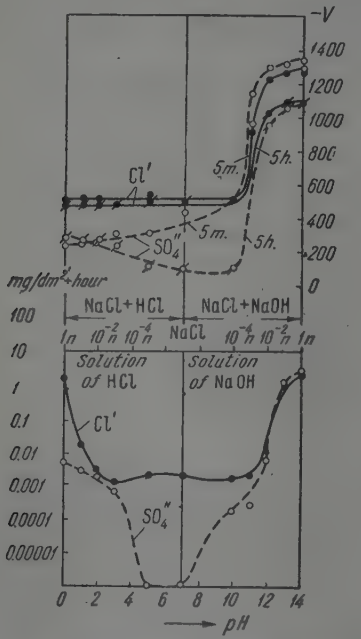


Fig. 1. Dependence of rate of corrosion and of electrode potential of aluminium on pH of electrolyte for 1 n solution of NaCl + HCl (or NaOH) and 1 n. Na_2SO_4+H_2SO_4 (or NaOH)

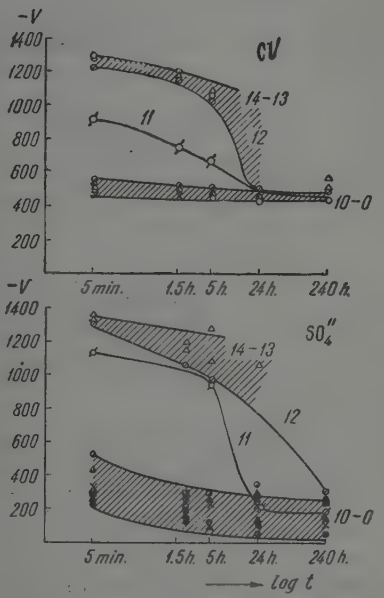


Fig. 2. Change of electrode potentials of aluminium with time in chloride and sulphate solutions with different pH values (marked on the curves)

ves of the dependence of electrode potentials on the pH of the solution for the same anions and for two moments in the measurements (5 min. and 4 hrs.). Curves obtained in the presence of the NO_3' ion proved to be very near to results obtained in sulphate solutions.

The conclusions to be drawn from the results obtained are as follows.

1. The curve showing the dependence of the rate of corrosion on the pH of the solution occupies approximately a symmetrical position in respect to the neutral region. In other words, both in the strongly alkaline region and in the strongly acid one the rate of corrosion is of the same order.

2. In neutral and acid solutions the rate of corrosion in the presence of the Cl' ion is much higher (in some solutions 1000 times higher) than in the presence of the SO_4'' ion. In strongly alkaline solutions the rate of corrosion in the presence of both ions is about the same.

3. In all acid, neutral and weakly alkaline solutions (up to pH = 10) the total (measured) electrode potential is positive (about—0.5 V for solutions of chlorides) in the course of corrosion. In more alkaline solutions corrosion proceeds at much higher negative values of potentials, reaching—1.3 V in the case of pH = 14.

4. In the presence of the SO_4'' ion the electrode potentials in the acid and alkaline regions are by far more electropositive than in the presence of the Cl'

ion. In the strongly alkaline region (above pH = 10) the difference is gradually diminishing with increase in alkalinity.

The variation of the electrode potentials with time is expressed on Fig. 2 in semi-logarithmical coordinates. It may be seen that for some pH values (0—10)

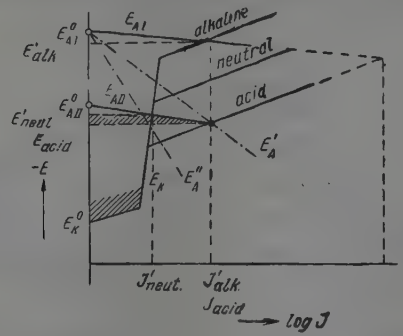


Fig. 3. Evans polarization diagram explaining differences in the values of the potential of aluminium in the alkaline region and in the acid and neutral regions at the same rates of corrosion in strongly acid and strongly alkaline regions

the potentials lie stably in the positive region; for strongly alkaline solutions (pH = 13—14) the values of the potentials are stable in a strongly negative region corresponding to an active aluminium. For intermediate values of the alkaline region (pH = 11—12) the potentials are unstable—they change from negative values (the more negative, the higher is the pH) to positive values characteristic of the neutral region. Special experiments have demonstrated that this change of the potential in the positive direction is not connected with a change in the composition of the solution, *i. e.* with the using up of the alkali during the corrosion process or with the absorption of carbon dioxide from the air * (though care was exercised to avoid this in experiments with alkaline solutions).

The lower values of the rates of corrosion in solutions containing the SO_4'' ion compared with solutions of chlorides can be explained by the ion SO_4'', having a much lower penetrating power (towards protective films) compared to the Cl' ion ([1]).

The following polarization diagram (Fig. 3) may be suggested to account for the interesting fact that in spite of the striking difference in electrode potentials between the strongly acid and strongly alkaline region, the rate of corrosion is of the same order of magnitude in both cases. The electrode potentials are plotted here against the logarithms of the electric current of micropairs on the surface of the corroding metal. The cathode curves are broken lines. Their first part corresponds to the electrochemical process of formation of OH' ions; the second (rising sharply upwards) corresponds to the process controlled by the diffusion of oxygen towards the cathode; and the third part is a hydrogen overvoltage curve. Variation in pH will cause the second part of the curve to shift but slightly, whereas the third part (hydrogen overvoltage) will be displaced very considerably. The higher the pH, the more negative is the region in which lies this segment of the cathode curve. The electric current may be as strong in the very acid as in the very alkaline region because of a shift in the value of the initial electrode potential of anodic areas (E^0_{AI}) in the positive direction for the acid and neutral region (E^0_{AII}), or because of a change in the slope of the anode curve (E'_A and E''_A, for the acid and neutral region).

* With more detail this process has been studied in our laboratory by E. N. Paleolog.

If the former is the true reason, the shift of the initial potential may be supposed to be connected with a special behaviour of adsorbed oxygen on the surface of the metal. On the other hand, a change in the slope of anode curves—if the reason should lie here — can be explained by a sharp decrease of the anodic area for the acid region and still more so for the neutral region due to a corresponding increase of the stability of the protective film.

Further experiments are being staged in order to check the explanation proposed.

Institute for Physical Chemistry.
Academy of Sciences of the USSR.

Received
28.III.1945.

REFERENCES

[1] Ю. Р. Э в а н с, Коррозия, пассивность и защита металлов, 1941, стр. 100.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

PHYSICAL CHEMISTRY

THE INFLUENCE OF VOLUME SORPTION ON THE MECHANICAL PROPERTIES OF POLYMERS

By S. N. ZHURKOV

(Communicated by A. F. Joffé, Member of the Academy, 16. XII. 1944)

In the present work a study is made of the elastic properties of high polymers with the help of the volume sorption of vapour. The molecules of volatile solvents absorbed by the polymer are used as test-particles to elucidate the interaction between the chain-like macromolecules in the process of mechanical strain. The experiments were carried out with fibres of acetate silk. The influence of absorption on the elastic properties was studied with single fibres 20 μ in diameter and one meter long, each cut into two parts, one of which was used for the mechanical measurements and the other for a parallel determination of the quantity of absorbed vapour with the help of a sensitive quartz balance. Spring balances made of a thin quartz thread made it possible to work with a very small load.

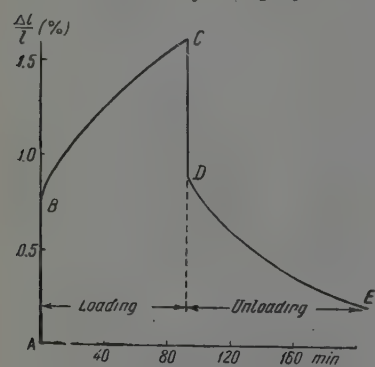


Fig. 1

Two balances, one loaded with the piece of fibre intended for absorption measurements, rolled up into a ball, and the other with the piece of fibre for the mechanical tests, were placed inside a special apparatus which was then evacuated and heated to 80—100°C. This removed the volatile substances sorbed by the silk, and after the fibre had been brought to constant weight, an experiment was begun.

The apparatus was filled with vapour, and after sorption equilibrium had set in, the deformation of the fibre and the absorption were measured. The concentration of the sorbate was controlled by the vapour pressure in the apparatus. When an experiment with one substance was over, it was carefully removed, and replaced by another (this method made it possible to carry out a series of experiments with the same fibre). Control experiments showed that repeated sorption and desorption does not alter the mechanical properties of the fibre.

Under the given conditions acethyl-cellulose fibre with 28 per cent of it hydroxyl groups non-esterificated was stretched by a constant load. The change in relative elongation as a function of the time is shown in Fig. 1. This time dependence, as is known, results from the superposition of the highly elastic and plastic deformations, taking place in a polymer under the action of an external force ([1],[2]).

The segments AB and CD on the elongation—time curve (Fig. 1) correspond to the most rapidly established elastic strain and characterize the ordinary elasticity of a solid body. As a matter of fact, Hooke's modulus for dry acetate silk computed from the initial elongation jump was found to be equal to 700 kg/mm². This is in fairly good agreement with the experimental data of Karger and Schmidt (³), who measured the elasticity modulus at the temperature of liquid air, where high elastic and plastic deformations are excluded.

The change in deformation during absorption makes it possible to follow the influence of the sorbate on the elasticity modulus. Experiments were carried out with the vapour of methyl, ethyl and butyl alcohols, water and acetone. Measurements were confined to relatively small concentrations (10—15 per cent), at which regularities of the simplest type could be expected.

The experiments have shown that the elasticity modulus of the fibre diminishes in proportion to the amount of material absorbed. The data obtained are plotted in Fig. 2, giving a series of straight lines for the different sorbates, each, as can be seen, having a different slope.

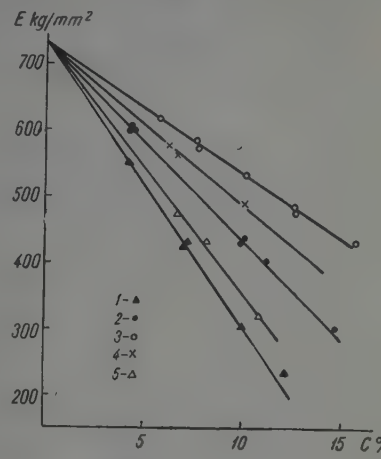


Fig. 2. *1*, methyl alcohol; *2*, ethyl alcohol; *3*, butyl alcohol; *4*, acetone; *5*, water

It would appear that the elasticity should be affected by the properties of the adsorbed molecules—their size, shape and the volume they occupy in the absorbed state. An analysis of the results shows, however, that the observed effect of decrease in elasticity does not depend upon the above-mentioned parameters, being determined solely by the number of sorbed molecules. As a matter of fact, sorption of equimolecular amounts of methyl, ethyl and butyl alcohols and acetone causes the same decrease in the elasticity modulus. The diverging lines in Fig. 2 merge into a single one if a quantity proportional to the number of absorbed molecules C/M (where M is the molecular weight) is plotted as abscissa instead of the concentration by weight.

Thus, the observed variation in the modulus is described by the equation

$$E = E_0 - k\frac{C}{M}$$

where E and E_0 are the elasticity modulus of the fibre in the dry and swollen states, respectively, and k is a constant.

Its mean value is $1.38 \cdot 10^3$, if E is expressed in kg/mm² and the concentration in g-mols per 100 g of acethyl cellulose. That k is very nearly constant, indeed, may be seen from the table.

It is also seen from the table that the value of k for water, which departs abruptly from the established rule, will within the limits of experimental error be of the same order of magnitude as the values of k for the other substances, if the molecular weight of the absorbed particle is doubled.This is tantamount to assuming that selective absorption of free associated water molecules present in the vapour near room temperature occurs here.

An equal change in elasticity at equal molar concentrations is also observed for the torsion modulus. The latter was determined from the period of torsional oscillations of a load suspended from the fibre. The relative change in the modulus for silk acetate during the absorption of ethyl alcohol and acetone is shown in Fig. 3.

Absorbed material	M	$k \cdot 10^{-3}$
Methyl alcohol	32	1.4
Ethyl »	46	1.38
Butyl »	74	1.38
Acetone	58	1.39
Water	18	0.685
»	2 × 18	1.37

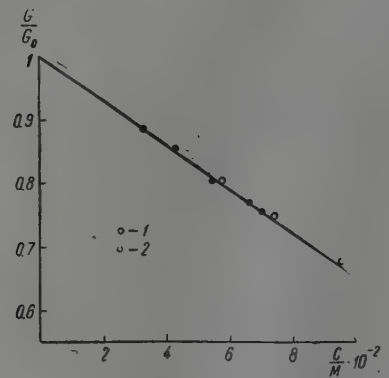


Fig. 3. Dependence of the torsion modulus on the molar concentration; *1*, ethyl alcohol; *2*, acetone

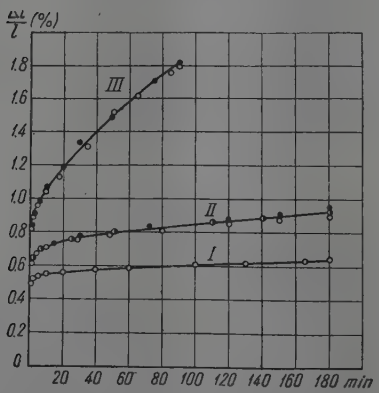


Fig. 4. Dependence of deformation on the time. *I*, dry fibre; *II*, methyl, ethyl and butyl alcohols in concentrations of 0.1 g-mol. in 100 g of acetyl cellulose; *III*, the same alcohols in concentrations of 0.22 g-mol. in 100 g of acetyl cellulose

In subsequent experiments it was established that the equimolecular law found for small elastic strains still holds for large deformations, involving appreciable displacements of the polymer chains with respect to one another.

Although in the latter case the dependence of the modulus upon the amount

of the sorbate is more complex, the elongation—time curves become identical when equimolecular quantities are absorbed by the fibre (Fig. 4).

These results can be explained as follows. The dipole groups of the molecules penetrating the fibre interact with the polar radicals of acetyl cellulose. The adsorbed molecules screen and neutralize the electrical field of the polar groups of the polymer. As a result, the molecular cohesion between the polymeric macromolecules is weakened, and the elasticity modulus decreases ([4]). From this point of view each sorbed polar molecule, irrespective of its chemical nature, should block one active group in the polymer. In the case of acetyl cellulose it is most probable that the molecules of water, acetone and alcohols are adsorbed on the non-esterificated hydroxyl groups ([5]).

The simple regularity observed makes it possible, we believe, to use absorbed molecules as test particles to elucidate intermolecular reactions in polymers. Making use of the selective nature of the sorption, we may hope to establish the rôle of individual functional groups of a polymeric molecule in the process of mechanical deformation.

Physical-Technical Institute.
Academy of Sciences of the USSR.
Leningrad.

Received
16.XII.1944.

REFERENCES

[1] P. P. Kobeko, E. V. Kuvshinsky and G. I. Gurevich, Bull. Acad. Sci. URSS, sér. phys., **3**, 329 (1934). [2] J. Press. J. Appl. Phys., **14**, 224 (1943). [3] J. Karger u. Schmidt, Z. techn. Phys., **6**, 124 (1925). [4] S. N. Zhurkov and R. I. Lerman, C. R. Acad. Sci. URSS, XLVII, No. 2 (1945). [5] W. Baker, C. Fuller and N. Pape, J. Am. Chem. Soc., **64**, 776 (1942).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

CHEMICAL TECHNOLOGY

MULLITE REFRACTORY FROM A MIXTURE OF KAOLIN AND ALUMINA

By P. P. BUDNIKOV, Corresponding Member of the Academy

It has been established by a number of authors that kaolinite, which is the main constituent of clays and kaolins, undergoes transformations in the range of 950 to 1100°C. A mullitization process takes place whereby silica is liberated in the form of cristoballite (1).

This process can be represented as follows

$$3[Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O] = 3[Al_2O_3 \cdot 2SiO_2] + 6H_2O$$
$$3[Al_2O_3 \cdot 2SiO_2] = 3Al_2O_3 \cdot 2SiO_2 + 4SiO_2$$

The yield of mullite and the growth of its crystals increases with temperature, but the theoretical amount is never attained. This is because part of the alumina enters into combination with the melts of the clay or kaoline to form a silica glass of varying viscosity, according to the chemical composition of the glass and the temperature.

The formation in a clay refractory of a more or less viscous vitreous phase has an unfavourable effect on its deformation under a load at high temperatures (the time when the deformation sets in and the character of the deformation curve are affected).

In order to increase the range of deformation in this material while under load, as well as its thermal and chemical stability one has to eliminate the harmful influence of the vitreous phase. To a certain extent this can be achieved through introduction into the charge of materials high in alumina such as bauxite, diaspor, sillimanite, kyanite (disten), andalusite or alumina.

The best results are obtained by adding alumina, which at a certain temperature combines with the silica liberated from the kaolinite to form mullite crystals, according to the following scheme

$$Al_2O_3 \cdot 2SiO_2 + 2Al_2O_3 = 3Al_2O_3 \cdot 2SiO_2$$

As initial material the Prosiansk kaoline and commercial alumina can be used; their chemical composition in per cent is given in Table 1.

Table 1

Materials	Al_2O_3	SiO_2	Fe_2O_3	TiO_2	CaO	MgO	K_2O	Na_2O	SO_3	Ignition loss
Prosiansk kaolin (USSR)	37.39	46.17	0.39	0.37	0.59	0.08	0.36	0.29	0.09	19.69
Commercial alumina .	95.06	1.05	0.03	—	1.21	traces	0.29		0.19	1.20

An intimate mixture of fine-ground kaolin and alumina (wet mixing) in the ratio $Al_2O_3 : SiO_3 = 3 : 2$ was made into briquettes which were then burned at

1600, 1650 and 1670°C for 2 hours. The burnt samples were subjected to a petrographic investigation and analysed for mullite quantitatively by means of treatment with 20 per cent H_2F_2.

Their thin-sections under the microscope showed a homogeneous mass of mullite crystals. The amount of the crystalline component was greater, and the crystals were larger in the samples burned at 1670°C.

Quantitative estimations gave the following results:

Burning temperature, °C	Content of mullite, %
1600	75.2
1650	81.4
1670	83.7

Besides mullite there were always a certain amount of corundum (6—10 per cent) and silica glass in the product. A 2—4 per cent addition of such substances as cryolite, fluorite, $MgCl_2$, P_2O_5, B_2O_3, *etc.*, will lower the burning temperature, whereby the viscosity of the glass is decreased and the crystallization of mullite accelerated.

From a kaolin-alumina mixture added with 2 per cent $MgCl_2$ a refractory was prepared of the following granulometric composition: grains under 0.5 mm, 30 per cent; from 0.5 to 1 mm, 45 per cent; from 1 to 2 mm, 25 per cent.

For binding material the PB clay from Chasov-Yar was used (12 per cent). The obtained mass (moisture 6.5 per cent) was pressed into cylinders 5 to 5 cm, which were burned at 1500°C.

These samples were tested for deformation under a load of 2 kg/cm² at high temperatures. In Table 2 the results of the tests are compared with the data for a semi-acid refractory of high quality ([2]), for an ordinary refractory and for kaolin refractory ([3]).

Table 2

	Start of deformation	Deformation 4%	Deformation 40%
Mullite refractory from kaolin and alumina	1520°	1560°	1590°
Kaolin refractory	1400°	1480°	—
Ordinary refractory from Chasov-Yar clay	1320°	1450°	1520°
Semi-acid refractory of high quality	1350°	1440°	1590°

It appears from these tests that the mullite refractory prepared from a close mixture of kaolin and silica is superior, as regards deformation under load, to both the ordinary refractory and kaolin refractory.

Received
21.IV.1945.

REFERENCES

[1] П. П. Будников и Б. А. Хиж, ЖПХ, № 1, 21 (1930); П. П. Будников, Огнеупоры, № 4, 250, 255 (1935); он же, ЖПХ, №5, 607 (1931); он же, Американская техника и промышленность, X, № 7, 213 (1933); В. В. Гончаров и А. Т. Клейнберг, Огнеупоры, № 4, 1192 (1938). [2] П. П. Будников, ЖПХ, № 7, 1153 (1934); P. P. Budnikov, Ceramic age (USA), **6**, 165 (1934). [3] П. П. Будников и В. П. Ендовицкий, Сталь, № 1—2, 802 (1933).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

GEOLOGY

CERTAIN QUESTIONS INVOLVED IN THE GEOLOGY OF THE UPPER PERMIAN DEPOSITS OF THE MIDDLE PART OF THE NEAR-KAMA REGION

By VITALY PEREDERIEV

(Communicated by V. A. Obruchev, Member of the Academy, 30. IX. 1944)

In connexion with findings of native mercury (2), cinnabar, wolframite, scheelite, cassiterite, gold, basobismuthite, bismuthite, molybdenite and zircon in the black sands of the sedimentary deposits of the basin of the Chernaya River in the middle part of the near-Kama area, came to the fore the investigation of the geological environment, age relations and palaeogeographical past of this region, which had not been discussed in detail since the time of Stuckenberg(4). It appeared quite imperative to retrace the conditions under which took place the formation of the individual geological complexes and minerals.

On the basis of field examination and a detailed laboratory treatment (carried out with the aid of methods of pollen, granulometric, chemical, black sand, microscopic, thermal spectrum, mercurimetric-nephelometric analyses) of a rich material collected in the course of prospecting in the «Gladky Mys» area and of an itinerary geological survey in Upper Permian deposits of the basin of the Chernaya River, the following geological generalizations could be reached.

The terrigenous complex of rocks of the red-coloured formation of the Kazan stage (deposits parallel to Zechstein) consists of two members: a red sandstone-argillite member—the Yug (Lower Kazan) series up to 60 m thick, and a grey argillite-conglomerate-sandstone member—the Belebey (Upper Kazan) series up to 100 m thick.

The red sandstone-argillite beds of the Yug (Lower Kazan) series ($P_{2kaz}^{jug\ (a)\ and\ (b)}$) are composed mainly of brown and red-brown compact microbedded platy argillites, occasionally green, dark-grey, variegated, banded, mottled, arenaceous, calcareous, bituminous areas, with lenticular accumulations. Aside from these there are also thin unevenly distributed layers of grey and brownish-grey, uneven-grained, clayey, calcareous, conglomerate-like, volkonskoitized sandstones, cross-bedded over some areas at various levels. Organic remains: natural casts and detritus of shells of *Anthracomya* sp. and *Palaeomutela* sp., fragments of spicules of *Silicispongiae*, accumulations of small shells of *Ostracoda*, impressions of leaves of *Psygmophyllum* ex gr. *expansum* Schimp., stems of *Dadoxylon biarmicum* Kutorga, and other vegetable remains difficult to determine, and lignite.

The grey argillite-sandstone beds of the Belebey (Upper Kazan) stage ($P_{2kaz_2}^{bel}$) represent an uneven interbedding of grey and brownish-grey, uneven-grained argillaceous and calcareous sandstones, in some areas cross-bedded, as well as of brown, occasionally mottled, banded, arenaceous, calcareous, dolomitized platy argillites and pebbly conglomerates with lenticular accumulations of pebbly conglomerates at various heights, and of grey, brownish-grey, uneven-grained calcareous conglomerate-like sandstones. Organic remains: accumulations of small shells of *Ostracoda*, scarce carbonaceous and vegetable remains difficult to determine (fragments of tissues); in pebbles of jaspers and lydites there are spheroids of *Radiolaria*.

In order to denote the dense argillaceous and variegated varieties of colloidal and metacolloidal Upper Permian deposits, which are widespread here particu-

larly in red beds, the term «argillite» has been introduced instead of the vague local names—«vap», «rukhlyak» (rottenstone), and the incorrect names of «red clay», «marl», *etc.* A syngenetic origin of the green colour of the argillites and an epigenetic origin of the red and brown colours have been established. The green colour and its various shades are characteristic of chlorite, while goethite is brown, red and of intermediate hues. As the green chromophore serves bivalent iron; as the red and brown chromophore, trivalent iron (3).

The sandstones are polymikt, of fluvial deltaic origin.

The chief geological differences and points of resemblance between the sandstones of the red and grey members of the region are given in the table.

The pebble complex of conglomerates of the grey beds of the following polymikt petrographic composition: quartzites, lydites, variegated jaspers, pyroxenic andesite-porphyrites, white quartz—closely resembles in its macroscopic habit and microscopic structure the Middle Palaeozoic jaspers and Upper Palaeozoic porphyrites of the North and Central Urals. The chief rock-forming ingredients of the pebbles are: quartzites, lydites, jaspers. In the pebbles of lydites and jaspers have been discovered numerous spheroid globular skeletons of *Radiolaria*. The conglomerates are of a fluvial, deltaic origin.

The stratification of the conglomerates consists of lenses and beds. Thickness of lenses 8—10 m. Cement: argillo-calcareo-arenaceous, argillo-areno-calcareous, calcitic, 20 to 40 per cent. Copper oxide in the cement of conglomerates is absent; chromium oxide is absent in the cement of the middle and upper horizons of the grey beds. The filling substance is formed by cross-bedded sandstones 20—30 per cent.

Size of the largest pebbles, 40 to 50 mm. The prevailing shape of the pebbles is an ellipsoidal or nearly spherical one; the surface is dull; the pebbles are arranged at random. Average volume of the pebbles, 1.46 cm^3; average diameter, 1.58 cm. Average coefficient of roundness, 82 per cent; average coefficient of flattening, 18 per cent. The content of black sand in the cement of conglomerates is insignificant; the black-sand complex of the minerals is similar to that of the sandstones and of the filling substance of the conglomerates of the grey beds, with the exception of cinnabar. The black sand content of the filling substance, the mineral association of the black sands according to the predominant minerals, the black-sand complex of minerals, the size of the rounded, tabular, platy grains of the black-sand minerals are similar to those of the sandstones of the grey beds. The grains of the black-sand minerals are mostly semi-rounded and rounded in shape. The allothigenic minerals (1 and 5) of the artificial black sands are: apatite, biotite, brookite, wolframite, hematite, garnet, gold, ilmenite, cassiterite, quartz, kyanite, cinnabar, magnetite, martite, microcline, molybdenite, oligoclase, orthoclase, hornblende, rutile, staurolite, sphene, titaniferous magnetite, tourmaline, chromite, zircon, scheelite + dolomite, calcite, clinozoisite, epidote. The genesis of the minerals is mechanogenic.

The authigenic minerals are: anatase, basobismuthite, bismuthite, vermiculite, volkonskoite, goethite, kaolinite, leucoxene, limonite, marcasite, pyrite, pyrolusite, sericite, coaly particles, chlorite + dolomite, calcite, clinozoisite, epidote.

The correlative minerals are: volkonskoite—in the red beds only: cinnabar—in the grey beds; wolframite and scheelite—an increased content and a larger size of grains in the grey beds, while in the red beds there are traces; chromite prevails in the red beds, while its content in the grey beds is small.

The grey argillite-conglomerate-sandstone beds of the Belebey (Upper Kazan) series are the chief source of cinnabar, wolframite and sheelite.

The following black-sand accessory minerals have been discovered in the sedimentary deposits of the middle part of the near-Kama region for the first time: cinnabar, wolframite, scheelite, cassiterite, gold, bismuthite, basobismuthite, molybdenite, brookite, vermiculite, marcasite, martite, pyrolusite, staurolite, titaniferous magnetite.

The source of the terrigenous complex of rocks of the red-coloured formation of the Kazan stage is the Palaeo-Urals with its complex variegated petrographic

Characteristics	Sandstones of the red beds of the Yug series $P_{2}^{jug}\ kaz_1$ (a) and (b)	Sandstones of the grey beds of the Belebey series $P_{2}^{bel}\ kaz_2$
Stratification	Lenses and strata	Lenses and strata
Thickness of lenses	Up to 20 m	Up to 20 m
Average thickness of strata	1.28 m	5.00 m
Varieties	Argillaceous, calcareous, volkonskoitized, conglomerate-like	Argillaceous, calcareous, conglomerate-like
Degree of cementation	High	Lower
Cement	Kaolinite, calcite, goethite	Kaolinite, calcite, goethite
Clay content	Lower	Higher
Lime content	Higher	Lower
Texture	Mainly fine-grained and medium-grained, cross-bedded	Medium grained (and coarse-grained), cross-bedded
Structure	Polymikt, aleuro-pelitic, uneven-grained with an epigenetic, monocrystaline (poikilitic) cement (structure of the Fontainebleau type)	
Granulometric composition, in indices	(091), (055)	(046), (064)
Predominant fraction	0.10 — 0.25 mm	0.10 — 0.25 mm
Shape of grains	Chiefly semi-rounded	Chiefly semi-rounded
Rock-forming minerals	Acid plagioclase, orthoclase, microcline, quartz, chlorite (clinochlore), biotite, goethite, epidote and clinozoisite, carbonaceous particles, *etc.*	
Black-sand content	Chief carriers of black sands of the terrigenous complex of rocks of the red-coloured formation of the Kazan stage	
Mineral association of black sand, according to predominant minerals	Magnetite-hematite-martite-epidote-chromite-zircon (B association)	

Characteristics	Sandstones of the red beds of the Yug series $P_{2\ kaz_1}^{jug\ (a)\ and\ (b)}$	Sandstones of the grey beds of the Belebey series $P_{2\ kaz_2}^{bel}$
Black-sand complex of minerals . .	Magnetite, titaniferous magnetite, amphiboles, wolframite, hematite, garnet, ilmenite, limonite, martite, staurolite, tourmaline, chromite, epidote, quartz, feldspar, calcite, chlorite, anatase, apatite, brookite, vermiculite, gold, cassiterite, kyanite, leucoxene, marcasite, pyrite, pyrolusite, rutile, sphene, zircon, scheelite	Same minerals as in red beds + cinnabar
Predominant shape of grains of black-sand minerals	Semi-rounded, rounded	Semi-rounded, rounded
Size of rounded grains of black-sand minerals	0.0 — 0.5 mm	0.01 — 0.5 mm
Size of platy, tabular grains of black-sand minerals	0.1 — 3 mm	0.1 — 3 mm
Cinnabar	None	Present
Chromite	Up to 30 per cent in the electromagnetic fraction	Up to 10 per cent in the electromagnetic fraction
Wolframite and scheelite	Isolated traces in the corresponding fraction	Up to 1 per cent in the corresponding fraction
Coarseness of wolframite grains . .	0.3 mm	1 mm
Volkonskoite	Present	None
Copper oxide	Present	Traces
Chromium oxide	Much	Little
Inclusions	Concretions of calcareous sandstones, pebbles of arenaceous and dolomitized argillites, rounded andi semi-rounded pebbles of quartzites, lydites, jaspers, of an Uralian habit, fragments of feldspar, epidote, quartz, dark-green volkonskoite	

habit: granitoid rocks, basic and ultrabasic rocks, ore deposits of tungsten, tin, gold, mercury, molybdenum, bismuth, metamorphic rocks, sedimentary rocks, weathering crust, placers of valuable and rare minerals. During the Yug time in the area of the Palaeo-Urals took place a denudation, most probably of the weathering crust of igneous and metamorphic rocks, with deposits of iron ores, chromite and copper, which is responsible for the specific habit of the Yug terrigenous mineralogical province. The erosion of the bulk of the weathering crust was followed in the area of the Palaeo-Urals during the Belebey time by denudation, chiefly of metamorphic and igneous rocks with deposits of rare metals: mercury, tungsten, bismuth, molybdenum, tin, gold (or partly their alluvial placers), which caused the specific habit of the Belebey terrigenous mineralogical province.

As agencies of transformation and deposition of clastic terrigenous material of the red beds served a hypothetical system of the Palaeo-Yug River (possibly of several large rivers), with a westerly, south-westerly course; in the case of grey beds it was the likewise hypothetical river system of the Palaeo-Belebey River (possibly, of several large rivers) with a south-westerly course.

The clastic sediments both of the red-coloured and of the grey-coloured deposits were accumulated: the former, in the Yug delta of the Palaeo-Yug River; the latter, in the Belebey delta of the Palaeo-Belebey River—in the littoral zone of the Zechstein sea, forming the Yug (Lower Kazan) and the Belebey (Upper Kazan) terrigenous mineralogical provinces; a more precise determination of their outlines is a matter of future geological studies. In the course of deposition mainly mechanical differentiation occurred in the deltas, *viz.* piling up of clastic organic and inorganic particles.

Owing to diagenetic and katagenetic processes from the clastic (partly colloidal) material were created argillites and sandstones of the red beds, argillites, conglomerates and sandstones of the grey beds, as well as genetically related accumulations of grains of valuable (and rare-metal) minerals, deposits of volkonskoite and brown coal, authigenic minerals.

To be expected are old deltaic (and fluvial) placers of wolframite, scheelite, zircon, especially in the sandstone stratigraphic units of the grey beds of the Belebey (Upper Kazan) terrigenous mineralogical province in the area of the adjacent eastern, north-eastern, and north-north-eastern regions of the middle part of the near-Kama area.

The erosion of the Upper Permian (chiefly Upper Kazan) fossil deltaic placers of wolframite, scheelite, zircon, cinnabar was the cause of the formation of local pocket-like placers of these minerals in the eluvial-diluvial-alluvial complex of rocks of the Quaternary deposits of the region.

In order to retrace with greater precision the outlines of the Yug and Belebey terrigenous mineralogical provinces and to shed additional light upon the problems of the economic significance of the placers of valuable (and rare) minerals of the Kazan and post-Pliocene ages, geological explorations should be continued and extended to the areas between the Siva and Kama rivers, as well as those in the left bank of the Kama. The data of field and laboratory investigations, the geological setting, the geological history of the region and of the adjacent regions of the middle part of the near-Kama area enable us to consider untenable the hypothesis of hydrothermal origin of valuable and rare-metal minerals of the black sands ([2]), cinnabar included.

The Sverdlovsk Mining Institute.
«Uralzvetmetrazvedka» Trust.

Received
30.IX.1944.

REFERENCES

[1] В. П. Батурин, Палеогеография по терригенным компонентам, 1937. [2] Б. А. Гаврусевич, Открытие ртути в Черновском районе и некоторые проблемы геологии и геохимии Молотовской области, отдельный оттиск, июль 1943. [3] А. Е. Ферсман, Цвета минералов, 1936. [4] А. Штукенберг, Общая геологическая карта России, лист 127, Труды ГК, XVI, № 1 (1898). [5] P. G. H. Boswell, On the Mineralogy of Sedimentary Rocks, London, 1933.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

GEOLOGY

TECTONIC STRUCTURE OF THE UFA PLATEAU ACCORDING TO THE LATEST DATA

By N. G. CHOCHIA

(Communicated by V. A. Obrucheo, Member of the Academy, 25. VI 1945)

The tectonic structure of the Ufa plateau has scarcely been treated in the recent geological literature. Yet, as a result of the work carried out by the All-Union Petroleum Institute, jointly with the Moscow Geological Trust and the Molotov Petroleum Combinate, new data have been obtained which give a somewhat new idea of this region.

The Ufa plateau is a gently sloping structure of the platform type, somewhat more distinctly pronounced than other structures of this type, probably because of its proximity to the Urals. If we are to judge by the exposures at the surface of Artinskian and Upper Carboniferous deposits, the width of this plateau decreases from 90 km north of the Kara-Tau Range to 15 km on the Sylva River.

The anticline of the Ufa plateau is stretched in a direction approaching the meridional, and plunges gently to the north. In the south it is broken by the Kara-Tau overthrust. The axis of this fold strikes in a nearly meridional direction in the form of a huge arc convex to the east. It is of interest to note that its general strike coincides amazingly with the western boundary of the exposures of igneous and metamorphic series all along the western slope of the Urals.

On account of the fact that the thickness of the Upper Carboniferous and Artinskian deposits decreases at a rate of up to 5 m per km as they extend westward (which corresponds to an angle of incline of 5′), the axis in the deep horizons of the structure of the plateau is likely to be displaced westward, while the incline of the limb becomes very gentle. The angle of pitch of the bend of the fold as estimated by V. D. Nalivkin (1942—1943) south of the Sarau River, following the roof of the Yurezan limestones, and north of the river, following the roof of the Irghina limestones, has the following value: in the area of the Yurezan River—Sukhanka River, 6′; Log Sukhanka—Kartya Ravine, horizontal attitude; Kartya Ravine—Bargadja River, 7′; Bargadja River—Ai River, 10′; Ai River—Sarana River, 21′; Sarana River—Sylva River, 8′. The eastern limb of the anticline is rather steep, the western is gentler.

The angles of incline of the limbs have been estimated by several geologists throughout the Ufa plateau. The data obtained are tabulated below.

From the table it will be seen that the steepness of the limbs remains rather constant in the southward direction.

The angles of incline of the limbs increase away from the axis; the proximal part is strongly flattened.

Only a comparatively small area of the Ufa plateau has hitherto been covered with detailed geological and structural survey; the work achieved, however, has already revealed several secondary uplifts against the general background of the anticline. By now 13 such uplifts are known; several of them are located

Position of profile	Western limb	Eastern limb
Yurezan River	20 — 30′, lower course of the Yurezan (A. L. Kudlai)	2 — 3° (V. G. Machina); 3 — 5° (S. S. Osipov). Ataf River, 7 km from the axis
Koshelevka River . . .	—	1°10′ (V. D. Nalivkin) and near the axis (N. G. Chochia)
In the interval Kartya Ravine—Bargadja River	1°10′ (V. D. Nalivkin)	1°45′ (V. D. Nalivkin)
Ai River	40′ (V. D. Nalivkin)	3° — 3°15′ (N. G. Chochia) near the axis; 5° (N. G. Chochia), 3 — 5 km from the axis
Town of Krasnoufimsk .	—	2 — 3° (V. D. Nalivkin), following the base of the Kungurian deposits; 6° (V. D. Nalivkin), following the upper beds with fusulinids. Both values have been determined 15 km from the axis
Sarana River	20 — 40′ (V. I. Nosal), 25 km from the axis. From 4 to 15′ (N. V. Dorofeev), 12 km from the axis	From 20 to 34′ (N. V. Dorofeev), near the axis
Nizhne-Irghina Works .	6 — 10′ (V. D. Nalivkin)	18′ (V. D. Nalivkin), 2 km from the axis of Mokshino; 30′ (V. D. Nalivkin), 5 km from the axis of the N. Irghina Works
North of the Sylva River	20′ (B. I. Graifer)	1 — 2° (B. I. Graifer)

on the axis of the anticline and have been discovered by the method of routine geological survey. The other six uplifts are confined to the western limb and have been outlined by the method of structural survey. These uplifts have been described in detail by V. D. Nalivkin (1945), V. I. Nosal (1944—1945), A. L. Kudlai (1938), N. G. Chochia (1944), N. V. Seleznev (1937), N. V. Dorofeev (1944, 1945), and others. In the axial part of the Ufa plateau the uplifts are dome-shaped. Their area covers several scores of kilometres, the amplitude reaching 70 m. Their western limbs are steeper than the eastern ones. A detailed description of them has been made by V. D. Nalivkin (1945). On the western limb of the plateau structural noses are developed, having steeper western limbs.

According to data by A. L. Kudlai and V. I. Nosal, the shape of the uplifts is irregular and rounded here; their area varies within a range from 15 to 70 km², the amplitude reaching 100 m. The angles of incline of the limbs do not exceed 1° in the central parts of the uplifts; on the periphery they may occasionally attain 3—6°. Some of the geologists who had studied the Ufa plateau (M. M. Tolstikhina [3]; A. N. Nazaryan, 1932; M. V. Kruglov) described a fault stretching along its eastern border. G. N. Frederix and T. M. Emelyantzev[4], and G. I. Kirichenko (1930) have even described a series of faults and overthrusts.

In order to refute the statement made by G. N. Frederix as to the existence of step-faults in the Ufa plateau, it is sufficient to compare the geological map compiled by him on a scale 1 : 200 000 of the region surrounding the town of Krasnoufimsk, with the geological maps compiled on a scale 1 : 50 000 adjoined to the paper of N. V. Dorofeev (1944, 1945). In Frederix's map the areas of distribution of the various horizons are bounded by straight fault lines of a meridional strike. In the geological maps of N. V. Dorofeev they have outlines corresponding to the boundaries of the rocks lying at small angles, cut by a system of ravines and rivulets and traceable throughout the lines of faults mentioned. It is necessary to point out that none of the boundaries in the region of such «doubtful» places was drawn until test-pits were carefully sunk and the elevation marks of the points obtained were measured with the aid of instruments.

It is somewhat more difficult to prove the absence of a fault along the eastern border of the plateau, outcrops being practically absent within the belt. In the environs of the B. Klyuchi village, west of the region described, M. V. Kruglov ([1]) and G. N. Frederix ([4]) have drawn a fault along the valley of the Irghina River, on account of the fact that along the western bank up to a height of 40—50 m occur Irghina limestones which are absent on the eastern bank, where Lemaza breccia limestones are found. The width of the valley is about 1.5 km. The absence of tectonic disturbances could be settled with the aid of a drill-hole sunk on the eastern bank at the «Klyuchi» health resort. It has opened up Irghina limestones at a depth of 24 m. This made it possible to calculate the angle of incline of the eastern limb which in this area proved to be 2°20′—2°30′, and entirely to reject the presence of a fault.

As shown by the results of geological mapping, in the more southerly region all along the eastern border of the plateau the Lemaza breccia limestones pitch without disturbances in an eastward direction towards the Yurezan-Sylva depression. No exception to this rule has been observed. It should be pointed out, however, that in several places of the plateau steep angles of dip have been observed, which could not be related to any definite structure. They were observed by a number of explorers along the Yurezan River, on Mt. Kosoy Kamen, upstream the Ust-Atavka village, where the angle attains 20—30°, near the Tashulovo village (10°) and along the line of the Kazan railway, where at the 1406th km an angle of 30° has been measured, and at the 1407th km a small anticline is located.

A detailed analysis of these data will be the subject of our further investigation.

The All-Union Petroleum Institute.

Received
25.VI.1945.

REFERENCES

[1] М. В. Круглов, Тр. ВГРО, в. 279 (1933). [2] С. С. Осипов, Проблемы сов. геол., № 4 (1933). [3] М. М. Толстихина, Изв. ГГРУ, **51**, в. 63 (1932). [4] Г. Н. Фредерикс и Т. М. Емельянцев, Записки Минерал. о-ва, нов. серия, XI, № 1 (1932).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

HYDROGEOLOGY

HYDROGEOLOGICAL CONDITIONS OF STABILITY OF GAS AND OIL FIELDS OF THE VOLGA REGION

By A. B. RONOV

(Communicated by F. P. Savarensky, Member of the Academy, 30. IX. 1944)

A study of the geology and geochemistry of oil and gas fields of the Volga region and of the history of its edge waters enabled the author to form an idea as to the part played by certain hydrogeological and geochemical factors operative in the processes of decomposition of oil and gas accumulations and in the conditions that are favourable for their preservation.

The destiny of oil and especially of gas fields depended, in the first place, upon the thickness and stability of the gas-resistant horizons. If there were no sufficiently stable gas-proof horizons in the entire section, the processes of gas liberation from solution could not have brought about any accumulation of gas, and the gas molecules must have diffused through the rocks into the atmosphere. If they met on their way a stable gas-resistant horizon, gas accumulation took place under the latter. Secondary processes of weakening of gas-proof horizons, associated either with erosion (Teplovka) or with disjunctive fractures (Syzran), led to the migration of gas accumulations and to the separation of gas from the oil. Under the most stable Kungurian (P_1K) and Kazan (P_2^{kaz}) gas-resistant horizons in the northern and central regions of the Volga area the largest gas accumulations in this section were formed (Buguruslan, Tuymaza, Krasnokamsk, Polometz-Kokui field). In the south of the Volga region, where these horizons had been eroded, the clays of the Vereyan stage ($C_2^{v(r)}$) served as a barrier under whose protection were formed and preserved the largest accumulations.

No less importance in the process of formation of gas occurrences should be attached to the magnitude of the pressure and the porosity of rocks, which are responsible for the amount of gas which can be held in the occurrence. It should be pointed out that the porosity of rocks varies with time; under the influence of secondary processes associated chiefly with the interaction between edge waters, hydrocarbons and enclosing rocks, it may either decrease (secondary calcitization), or increase (secondary dolomitization and leaching processes). Pressure also varies in time, its change being determined by the increase of the anticlinal uplift and chiefly by the epeirogenic movements of the Earth's crust in the area of the gas occurrence and in the area of the head (1). The change of the height of the water column which determines the head of the edge waters invariably brings about a corresponding change in the pressure in the oil and gas fields.

Of essential importance in this connexion is also the specific gravity of water, which will be clearly seen from the equation $P = 0.1\,hd$, where P is the pressure in the bed, h is the height of the water column, and d is the specific gravity of water. It was shown that the specific gravity of the edge waters of the Volga region changed in the course of time.

The preservation of gas occurrences also depends to a large extent on the rate of their washing with waters. Where the edge waters move at a higher rate, a degasing of the deposits takes place, gas particles being carried away with the water and brought to the surface within the area of discharge. The slower the stream of water and the higher its mineralization. the smaller is the amount of gas which it is capable to dissolve and carry along. In the area of development of stagnant edge waters such a mode of degasing is excluded (North Volga region).

An increase in the mineralization of edge waters creates a medium less favourable to the activity of microorganisms, as may be seen from papers by V. P. Savchenko. In the Northern underground basin of the Volga region(1), whose brines approach salt saturation, mainly processes of denitrification are observed, which enrich the dissolved and free gases with biogenic nitrogen (Polometz-Kokui, Kukar-Ivkinsky deposits, Buldyr). The stagnant condition of the water leads to their enrichment in helium, in the accumulation of which a decisive part is played by the factor of time. Owing to this, in all the gas fields of the Northern underground basin a higher content of this exceedingly mobile element has been recorded; it shows that here during a considerable length of time the rocks have not been washed by young waters.

The stability of liquid and gaseous hydrocarbons depends also on the presence of calcium sulphates in the water and enclosing rocks.

The processes of a microbiological reduction of sulphates are associated with oil fields and accumulations of hydrocarbon gases. The edge waters of the north of the Volga region are enriched in calcium sulphates. A considerable quantity of gypsum and anhydrites is contained also in the enclosing rocks. In some oil and gas fields, however, the bulk of the hydrocarbons has been preserved intact, while in others a complete decomposition of the oil and gas occurrences has taken place. Thus, for instance, in the region of Baitugan, Buldyr and Teplovka the processes of decomposition were so intense and their rate so high that the section retained only some traces of hydrocarbon occurrences once present there, while at Krasnokamsk, Tuymaza, Buguruslan, Syzran and Yelshanka large oil and gas accumulations have survived.

If the process of sulphate reduction went on everywhere at the same rate, no economic occurrences of liquid and, so much the more of gaseous hydrocarbons would have anywhere survived by now. But the rate of sulphate reduction does not seem to be everywhere the same. In the first place it is controlled by the absolute content of sulphates in the water and the enclosing rocks. Thus, for example, sulphates are lacking in the section of Yelshanka deposit. They are practically absent also in the waters (0.6 m-equiv. per litre). This accounts for the fact that along with a relatively low mineralization of waters, great accumulations of hydrocarbon gases have been preserved here. It is also known that in the Vereyan horizon and the coal-bearing formation (C_1^{2h}) of the Krasnokamsk deposit great oil occurrences have survived, in spite of the fact that the absolute content of sulphates in the water exceeds that in the water of the Yelshanka deposit by one decimal place (12.6 m-equiv. per litre). The process of reduction of sulphates could have proceeded there for an infinitely great length of time, until the deposit would have been entirely decomposed, since the enclosing rocks contain an increased amount of calcium sulphates. An unknown cause has lowed down the rate of hydrocarbon decomposition. In the same field in the Artinskian rocks (P_1^a) the oil deposit has been decomposed and the oil converted to a residual asphalt-like mass. Similar relations have been reported for Tuymaza, where in the productive coal-bearing formation a great oil pool has been preserved, while directly under the Kungurian gypsum which here has been partly affected by Quaternary erosion, the oil pool has been destroyed. In both these fields the oil pools preserved are underlain by highly mineralized brines with a specific gravity of 1.17, while the decomposed pool at Krasnokamsk is underlain by less mineralized waters ($d = 1.09$), and by almost fresh waters at Tuymaza ($d = 1.03$).

An analysis of these relations brings us to the conclusion that highly mineralized brines approaching saturation are less favourable for the activity of de-

sulphurizing bacteria than slightly mineralized waters. For this reason the cumulative effect of their activity in highly mineralized waters is smaller, and the microbiological reaction sets in at a lower rate.

In the light of these conceptions the cause of the decomposition of the Vereyan oil pools at Buldyr, Baitugan and Teplovka becomes comprehensible. The erosion of the water-resistant horizons in these regions has brought about a decrease in the mineralization of the Vereyan edge water, and thereby produced more favourable conditions for bacterial activity. The exceptional degree of saturation of the terrigenous members of these deposits with pyrite, the calcite cement of sandstones, the richness of the edge waters in hydrogen sulphide, and also the fact itself of the oxidation of hydrocarbons, permits to state with full assurance that the processes of desulphurization proceeded according to the equation

$$CaSO_4 + CH_4 = CaCO_3 + H_2S + H_2O$$

The place of methane could have been taken also by heavier hydrocarbons. The hydrogen sulphide evolved as a result of the reaction combined with ferruginous minerals, forming pyrite, while the calcite filled up the fractures in the rocks and served as cement for the sandstones.

Analogous reactions proceeded in all the deposits of gaseous and liquid hydrocarbons of the Volga region, but because of the lower rate of the reaction, the ultimate effect was smaller, too. At Yelshanka, where the content of sulphates in the Vereyan and Oka waters is very low, the processes of desulphurization went on chiefly outside the gas-proof Vereyan horizon, within the Kashirian (C^k_2) and Jurassic rocks filled up with slightly mineralized sulphate waters. The methane molecules diffusing the gas-resistant clays, finding their way into Jurassic waters favourable for the activity of desulphurizing bacteria, were oxidized outside the primary gas accumulations. Some traces of the reaction are seen in the intense pyritization and calcitization of the Kashirian limestones and Jurassic clays.

The above said makes comprehensible why in most areas of the Volga region the Ufa sandstones (P_1^{uf}) contain oxidized oil. It has remained intact only where the accumulations of liquid and gaseous hydrocarbons were overlain by thick hydro-chemical gas-proof horizons of Spirifer age, which protected the edge waters from dilution (Buguruslan).

Institute of Theoretical Geophysics.
Academy of Sciences of the USSR.

Received
30.IX.1944.

REFERENCES

[1] A. B. Ronov, C. R. Acad. Sci. URSS, XLVIII, No. 5 (1945).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

CYTOLOGY

MITOTIC CELL ACTIVITY AS STUDIED BY EXPLANTATION OF THE CHICK EMBRYO HEART OF DIFFERENT AGES*

By A. F. IVANITZKAYA

(Communicated by A. A. Zavarzin, Member of the Academy, 28. VI. 1945)

It is well known that when added directly to explantation medium, or injected to animals employed as donors, thyroxin stimulates the migration and proliferation of fibroblastic cells ([1–3]). As to the effect of the thyroxin deficiency in explantation media, it has been but poorly studied up to the present day([4]). The present study is concerned with the second of the two problems mentioned above.

Heart tissue of chick embryos taken at different periods of incubation (from 4 to 20 days) was cultivated both in normal chicken plasma, and in the plasma of donors which had their thyroid glands extirpated at a previous date. Blood was taken from the experimental animals at different periods after the removal of their thyroid glands, but not until 5 days after operation. The heart tissue was cultivated after the classical method, in a hanging drop of plasma, without addition of embryonic extract. A total of 754 cultures were started. A large part of the cultures were fixed after 48 hours of cultivation. 10 per cent formaldehyde solution and Flemming's solution were used as fixing substances. Staining was done after Caracci's and Feulgen's methods. Explants taken from older embryos both in a normal and experimental medium were fixed somewhat later, since their latent period (from the start to the appearance of the first cells in the culture) was considerably longer. To evaluate the rate of growth of the cells Champy's mitotic coefficient was used which, in our opinion, is the most precise and objective criterion.

The growth of heart fragments of different ages has more than once been the object of observations. When determining the average daily increment of growing material from the fourth day of incubation and till the 35th day of postembryonic development, Olivo ([5]) has obtained an S-shaped curve of growth.

In his study of the growth of a whole embryo and of its separate organs Schmalhausen ([6]) found that the rate of growth of the heart could be expressed by a multimodal curve. Olivo and Slavich ([7]) made observations on the mitotic activity of cells, beginning from the second day of incubation up to the tenth day after hatching.

They found that two days after hatching the mitotic coefficient shows an abrupt drop down to zero by the tenth day.

Olivo and Gomirato([8]) who cultivated hearts of 3—7-day chick embryos under conditions varying in respect to hypotonicity found the response of the explants to differ according to age.

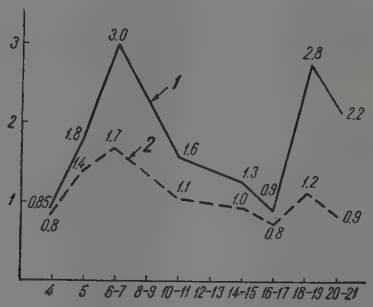
* The experimental part of the work was performed at the Institute of Experimental Morphogenesis, People's Commissariat of Education of the RSFSR.

Several methods are known for determining the rate of growth of tissue cultures [9—13].

In the present investigation Champy's method was adopted. Resting and dividing cells were counted.

The mitotic coefficient of the growing zone of explants taken from embryos of different ages varies greatly. In cultures of 4-day-old embryos cultivated in normal chick plasma the mitotic coefficient based upon counts of 4109 cells was found to be equal to 0.85 per cent. By the fifth day it was found to rise to 1.8 per cent. After 6—7 days of cultivation it equalled 3 per cent. From this time onwards the mitotic coefficient was found gradually to decrease, reaching its minimum, 0.9 per cent, after 16 days of cultivation. By the seventeenth day it rised again to 2.8 per cent, after which a second period of depression took place.

A similar course of age variation in mitotic activity was observed in cultures containing plasma of thyroidectomized animals. The respective values of the



Curves of mitotic coefficients of the zone of growth for embryos of different age, grown on normal or thyroidless plasma. *1*, normal plasma; *2*, plasma taken from thyroidectomized donors

mitotic coefficient for four, six-seven, sixteen, and eighteen days of cultivation were found to be 0.8 (based on 5349 cell-counts), 1.7, 0.8 and 1.2 per cent. After 18 days of cultivation a new depression of mitotic activity was also observed (see figure).

Thus, the mitotic activity was found to vary greatly both in normal plasma and in the plasma of blood taken from thyroidectomized donors, the variation being also dependent upon the age of the tissue. In cultures raised on normal plasma the variation of mitotic activity was considerably more pronounced than in the experimental cultures. The question arises as to the cause responsible for the high or low activity of the observed cells within the growing zone at the same periods of cultivation, both in experimental and control cultures. The most plausible explanation of the facts observed is to assume that cells of different types replace one another during the growth of the cultures. Observations on the growing cultures started from material of different ages have shown that myoblastic elements proliferate most actively in cultures from heart taken at the earliest days of incubation. They are clearly expressed and could be observed in a great part of the cultures. Fibroblastic elements are but rarely seen in the cultures at this period of development. The turning point in the behaviour of cultures is the seventh day of cultivation. From this day on the development of fibroblasts goes on more intensely, while myoblasts gradually decrease in number. Finally, in cultures of the heart of 18-day-old embryos fibroblasts are clearly predominating over myoblasts which are nearly absent in the cultures. This suggestion has further been corroborated by the increase in the mitotic coefficient which reaches by that time its second maximum both in the experimental and control series of the cultures. The curves of growth are clearly bimodal (see figure).

A comparison of the curves and of the mitotic coefficients of the cultures grown

on normal and experimental plasmas makes it evident that the curve of growth for the experimental (thyroidless) cultures is lying below that for the control series of the cultures, corresponding to the normal plasma. The absence of the thyroid hormone makes the plasma of thyroidectomized donors less active and less favourable for the cultivation of the heart of chick embryos.

Received
28.VI.1945.

REFERENCES

[1] S. Semura, Arch. exp. Zellforsch., X (1931). [2] J. S. Latta and J. Z. Davis, ibid., XXI (1931). [3] J. Verne et D. Odiette, C. R., 21 (1936). [4] S. Uchida, Biochem. Z., **163**, 75 (1936). [5] O. M. Olivo, Boll. Soc. Ital. Biol. Sper., 5 (1930). [6] I. I. Schmalhausen, Roux' Arch., **108** (1926). [7] O. Olivo e Slavich, Arch. Entwickl. Mech., 121 (1930). [9] O. Olivo e Gomirato, Boll. Soc. Ital. Biol. Sper. (1932). [9] A. Ebeling, J. Exp. Med., **34**, 231 (1921). [10] A. Fischer, Gewebezüchtung, 1930. [11] C. Champy, C. R. Soc. Biol., 76 (1914). [12] P. Gaillard, Arch. exp. Zellforsch., XI, H. 12 (1931). [13] P. Gaillard, Protoplasma, **24**, H. 3 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

GENETICS

TRIGENERIC HYBRIDS *ELYMUS* × WHEAT × RYE

By V. E. PISSAREV and N. M. VINOGRADOVA

(Communicated by N. V. Zyzin, Member of the Academy, 7. VII. 1945)

In the course of our work on crossing *Elymus* species with cereals, according to the suggestion of N. V. Zyzin, a number of crossings with rye-wheat amphidiploids were also made.

In 1942 we emasculated 556 flowers of the winter rye-wheat amphidiploid obtained under No.19 from the Institute for Grain Crops of the South-East.

Normal plants grown under ordinary field conditions without transplantation of embryos (1) were subjected to emasculation. The emasculated flowers were pollinated with pollen of *Elymus arenarius* L. A total of 11 grains were produced, two of which were small, but nevertheless fully developed. Only these two small grains germinated; germination was done in moist sterilized sand, in Petri dishes.

The hybrid plants were developing rapidly and produced ears toward the end of summer of 1943.

Thus, in spite of the presence of the two genomes of the winter type, the summer type of the *Elymus arenarius*, sample No. 19, proved dominant. Cytological analysis made in spring of 1944 at the cytological laboratory of this Institute (Poddubnaya-Arnoldi) revealed the expected chromosome number, *viz.* $2n = 56$, since the somatic chromosome number of the two parents was 56.

The tillering of the plants was slight. The hybrid produced, in contrast to the hybrid wheat × *Elymus* previously described by us, proved to be an annual one, and it could be perpetuated with difficulty by vegetative propagation during the year 1944 only; further growth could no more be obtained after the development of ears.

One should think that the two genomes of the annual crops (wheat and rye) present in the hybrid have suppressed the perennial habit of *Elymus*. The presence of two genomes bearing one and the same type of the ear affected the morphology of the ears in the hybrid. The ears of our first hybrid wheat × *Elymus* were of a mixed type, the spikelets being arranged after the wheat-type in the upper part of the ear (one spikelet per segment of the rachis), while the lower part was of the *Elymus*-type (2—3 spikelets per segment). Now the ears of the trigeneric hybrid appeared to be of the simple wheat-type.

Thus, the *Elymus*-type of the ear was here also suppressed by the two genomes of wheat and rye. The glumes of the hybrid were morphologically more like those of *Elymus*. The flowers were sterile in consequence of an underdevelopment of the anthers.

In 1943 we crossed the rye-wheat amphidiploid with another form of *Elymus arenarius*.

We have obtained here from 450 pollinated flowers 2 poorly filled seeds from which, however, trigeneric hybrids could be grown.

From crosses practised in 1944 between the rye-wheat amphidiploids with *Elymus arenarius* we obtained again in 1945 a few hybrids.

Crosses between the amphidiploids wheat × rye and *Elymus arenarius* are thus far more successful than the direct hybridization of wheat with *Elymus*.

The proportion of successful crosses is considerably higher in this combination; hybrid kernels are more readily obtained and can be brought to germination in the usual manner in sand sterilized by heat.

Institute for Grain Crop Husbandry.

Received 7.VII.1945.

REFERENCES

[1] V. E. Pissarev and N. M. Vinogradova, C. R. Acad. Sci. URSS, XLV, No. 3 (1944).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

HYDROBIOLOGY

COURSE OF EVOLUTION OF FLOOD PLAIN BASINS

By D. A. LASTOCHKIN

(Communicated by I. I. Schmalhausen, Member of the Academy, 4. VI. 1945)

As a result of his study carried out jointly with M. V. Zheltenkova, of the distribution and abundance of *Unionidae* in the lakes of the Volga flood plain near Kazan, the author arrived at the conclusion as to the necessity to revise

Scheme of flood plain segment of the Volga upstream of Kazan

the interpretation generally given to the course of evolution of flood plain basins ([1],[2]). According to this view, the oxbow lakes and ponds of a flood plain undergo regressive evolution, for it is assumed that the eroding and washing work of the overflowing streams and currents, operative at the time of seasonal overflows, is less efficacious than the cumulative effect of the agencies operative in an opposite direction, to which the basins are exposed during a much longer time interval between two overflows. The processes which take place in basins adjoining the edge of the first alluvial terrace have been but scarcely investigated if at all; the existence of basins is largely determined there by their being fed by the abundant ground waters, emerging at the surface in the form of rivulets, lakes and swamps.

Higher upstream of Kazan, near the village Vasilievo, the Volga makes a sharp turning to the right bluff, while a broad, ridged segment * of the flood plain is formed along the left bank of the river, which in the subsequent undergoes a sharp constriction at the approach to the Kazan barrier (see figure) which had once barred the river at this point.

It is no easy matter to retrace the location of the initial river bed which was formed upon the break of the barrier; to do so special investigation is necessary. At any rate it is hardly probable that the river rolled along the base of the left-bank terrace. It seems much more probable that it flew along the recent central zone of the flood plain which has still preserved the traces of the wan-

* At the flood plain surface of a large river, under the influence of the alluvial process, there is formed an uneven relief of alternating elevations and depressions; these elevations or ridges are slanty or low here.

dering of the river meanders in the form of several nearly parallel lake depressions, lying at an acute angle to the recent terrace of the river. Downstream the barrier the river which made its way through it at once found itself dammed by the Kama River, and the profuse sedimentation of the alluvial material soon transformed the ridged flood plain to an island flood plain.

As may be seen from the table, «young» lakes, deep, almost free from macrophytes (disconnected growths of *Polygonum amphibium* and *Potamogeton*), with a slightly silted sandy littoral, very high in *Unionidae* and with a diverse ichthyofauna, occur at various profiles of all possible zones of the alluvial plain, along with silty lakes overgrown with macrophytes (mortlakes), nearly destitute of *Unionidae* and inhabited by a monotonous ichthyofauna.

Lakes of Various Profiles of the Near-Kazan Flood Plain of the Volga River

Profiles	Zones of the left-bank flood plain		
	Near channel	Central	Near terrace
I. Observatorsky	Dubovoe, Gusinoe, very silty, strongly overgrown with macrophytes, with scarcely any *Unionidae* present	Bolshoe, slightly silty, free from macrophytes; 243.7 spec. of *Unionidae**	Sumka, Trostianoe, very silty, swampy, scarcely any *Unionidae* present
II. Zaimischensky	Morkvashinskoe, slightly silty or overgrown; 163.4 spec. of *Unionidae*	Krivoe, lower part medium silty or overgrown; 73.2 spec. of *Unionidae*	Bolshoe Glubokoe, slightly silty or overgrown; 268.0 *Unionidae* spec.
III. Judinsky	Arakchinsky back-water of sand-beach, medium siltiness, scarcely any growth; 63.5 spec. of *Unionidae*	Besymiannoe, medium siltiness or overgrowth; 80.3 spec. of *Unionidae*	Judinskoe, slightly silted, scarcely any growth; 84.6 spec. of *Unionidae*
IV. Shelanginsky		Besymiannoe, very silty and overgrown. No *Unionidae*	Klenovaya voloshka, slightly silty, scarcely any growth; 182 *Unionidae* spec.

According to Williams ([3]), the main feature of ridged flood plains are dependent upon the way the flood plain is penetrated and acted upon by the currents and flow of spring inundation. The water of the overflow penetrates through the upper part of the natural level of the flood plain segment, in the first line along the depressions of the relief: along the passages between lakes and river, through the lakes, between flood plain, *etc.* According to observations made by Zhukovsky ([4]) and Markovsky ([5]), the rate of the overflow current in the lacustrian depressions is twice that of the remaining tract of the flood plain. The suggestion made by Williams ([3], p. 326) as to the multitude of such streams of the flood land surface has still been underestimated (flood plain stream line which forms the thalwegs). Among these it is necessary to discriminate between the trunk line (the swiftest and strongest) and the secondary lines.

Plusnin ([6]) was right in pointing out that during seasonal overflows the water flows over the flood plain not as a single, powerful line, but as a multitude of separate streams or currents, each of which runs at a different velocity and in a different direction. The main or trunk line, or lines, if there are many, cause such an intense wash and erosion in the lakes situated along

* Figures characterizing the occurrence of *Unionidae* have been borrowed from the paper published jointly with M. V. Zheltenkova.

the axis that these oxbow lakes remain in a stable condition for a long time, or do even advance in their development, since the erosional work of the stream (removal of silt, wash of the banks and of the littoral, destruction of littoral macrophytes) proves to outbalance the work of the opposite agencies which are operative during the remaining time.

These lakes are distinguished by a narrow, rapidly falling littoral, and by the development of morphological features similar to those of river channels([5]). Their eroded banks are destitute of any woody vegetation, the upper end of the lake is bordered with sandy deposits, while laterally is developed a system of ridges and hollows parallel to the axis of the lake. For this type of lakes Beklemishev ([7],[8]) has suggested the name of «rejuvenated» or secondary ones, whereas Markovsky termed them river-bed lakes. As an example of the stability of the outlines of such lakes, Williams has described some of them from the flood plain of the Zna River, where they have been known to persist during 150 years.

Between the stream lines the overflow current is much less intense, and may die out almost altogether, even in the lower course of large rivers ([9],[10]). It has been convincingly demonstrated by Egereva that in such regions of the near alluvial terrace zone of the Lower Kama both physical-chemical conditions (transparence, contents of salts, oxidability) and composition of the plankton (a considerable admixture of pond and occasionally also of swamp forms) differ markedly from what is observed at corresponding periods in the main river channel.

At the point where the river valley undergoes abrupt constriction, overflow stream lines must join together, and the swiftness of the current increases; this results in an intensified erosional and washing work, affecting the flood plain and its lakes. As pointed out by Elenevsky([11]), the lakes become deep and clear within such regions of the Oka River owing to drainage; all the peculiar features of a near-terrace flood plain may be obliterated here.

These data make it possible to account for the apparent lack of regularity in the distribution of lakes according to the various profiles of the Near-Kazan flood plain. As may be seen from the schematic drawing attached to this paper, the flood plain streams which make their way through the upper end of the segment along the lake passages, must necessarily rush into the central zone of the flood plain, the more so as the Bolshoe lake discharges its water into the Volga River just in this end of the segment. The lakes in the near-channel zone of the first profile are protected against the powerful work of the streams by a high bank level which the Volga cannot surmount until the trunk lines have been formed. In the second profile, owing to the constriction which begins there, part of the flood plain thalwegs, especially those from lake Bolshoe, rush to the near-terrace zone; in spite of its being densely overgrown, lake Glubokoe shows a remarkably clean sandy littoral.

The remaining, probably minor streams rush from lakes Gusinoe, B. Goreloe, M. Goreloe, partly also from lake Bolshoe and along the lower part of lake Krivoe, into lake Morkvashinskoe, situated near the bank level. The third profile, the one corresponding to the abrupt constriction of the food plain, forms well-drained lakes in all the three zones. The fact that the Arakchinsky back-water and Judinskoe lake are rather poor in shells should be put down to the pollution of those water basins: the pure, terminal part of Judinskoe lake is characterized by a much higher occurrence of *Unionidae* (about 180—200 individuals per one metre of its shoreline).

The fourth profile should be referred to a different type of flood plain, *viz.* to the island type; the whole flood plain consists there in one, or several islands which had once been formed in the river bed and grew subsequently attached either to the continental bank, or to one another. Strictly speaking, there is no near-terrace zone in this case, the main volume of high water rushing into the passage between the terrace and the oldest, or unique, island. Such passages are known by the local name of «voloshka». Indeed, notwith-

standing the length and sinuosity of the path, and the distance separating such a passage from the recent channel, it is intensely washed by spring water. On the other hand, the central and near-channel zones are either entirely destitute of lakes, or else these are very small and subject to abundant overgrowth, owing to which they are but insufficiently washed.

Shelanga island (profile IV) which is 3 km across, in its portion most distant from the channel, is overgrown with broad-leaved woods; spring high water does nevertheless intensely erode the passage called Klenovaya voloshka which separates this island from the first terrace. The banks of the passage are sandy, steep, the littoral is very narrow, while its lower end is furrowed with current ripple-marks, which testify to an intensity of load traction similar to that of the recent river bed.

Within the region of the main thalweg of the broad portion of the ridged segment of the flood plain the lakes are thus found to persist in maintaining their juvenile condition during a long period of time. And this is still more true of the passages («voloshkas») of the island flood plain. In the narrow districts of the flood plain the lakes are likely to develop progressively and to approach to the fluvial type. The formation anew of large structures at any zone of such a narrowed flood plain does not seem to be out of the question.

Markovsky has demonstrated the existence of such neoformations, approaching the size of lakes, in the near-channel flood plain of the Dnieper River, the formation of which was due to the approach of the river to the flood plain; the same has been demonstrated by Beklemishev[7] for the Upper Urals. Plusnin has shown that neoformations of this kind are of especially frequent occurrence in the flood plain of the Lower Volga, where they are highly diverse. My personal observations made in the Syzran flood plain of the Lower Volga went to confirm the above evidence. And this is but natural besides, if we are to take into account the increase in the intensity of the annual overflow downstream a large river, and in the length of the period of the overflow.

Of great importance is the influence of abundant and especially of catastrophic inundations which may utterly change the direction of the evolution even of almost obliterated flood plain ponds. We know but little of the course of events taking place in near-terrace zone of ridged flood plains, where along with a rivulet and swamps there exist also true lakes. According to Williams, it would be a mistake to imagine that in this case there is operative a regressive process only, leading ultimately to the formation of lowland swamps.

The regressive evolution of flood plain basins in ridged and island flood plains is associated with changes in the paths of thalwegs. The wandering of a river bed within a meander belt, the barring of the actual way by an insurmountable obstacle due to the settling of a large amount of sandy or clayey sediments, and other causes may serve as an impetus for a continuous regress of the oxbow lakes (true mortlakes), which may even result in an ultimate stage of hydrosere unless a catastrophic overflow takes place during this period. The latter may inhibit, arrest this process, or even carry back the lake or the pond to the initial stage of evolution.

Received
4.VI.1945.

REFERENCES

[1] V. I. Schadin, Arch. Hydrobiologie, **35** (1932). [2] В. И. Жадин, Тр. Зоол. ин-та АН СССР, V, в. 2 (1940). [3] В. Р. Вильямс, Почвоведение, 2 (1926). [4] Н. Н. Жуковский, Издания н.-и. Управления НКПС, в. 7 (1930). [5] Ю. М. Марковский, Тр. Гидробиол. ин-та АН УССР, № 17 (1938). [6] И. В. Плюснин, Почвы Волго-Актюбинской поймы, 1938. [7] В. Н. Беклемишев и Ю. Г. Митрофанова, Изв. Биолог. н.-и. и-та Пермского ун-та, IV (1937). [8] В. Н. Беклемишев, А. А. Брюханова и Н. К. Шипицына, Предпосылки к эпидемиологии и профилактике малярии в Магнитогорске, 1931. [9] А. Н. Шенников, Луга Симбирской губернии, 1919. [10] Егерева, Учен. зап. Казанск. ун-та, **95**, в. 8 (1935). [11] Р. А. Еленевский, Учен. зап. Горьковск. ун-та, **5** (1931).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

PLANT PHYSIOLOGY

EFFECT OF RADIOACTIVE ELEMENTS UPON DEVELOPMENT OF THE ROOT-NODULE BACTERIA AND UPON THE ASSIMILATION BY THEM OF THE MOLECULAR NITROGEN OF THE ATMOSPHERE

By A. A. DROBKOV

(Communicated by D. N. Prjanischnikow, Member of the Academy, 28. IV. 1945)

In our experiments on peas ([1]) conducted in 1941, and earlier, we have noticed root-nodule formation in plants grown in a medium containing radium, while in the control cultures grown in the absence of radium no formation of nodules was observed. No inoculation being practised, the bacteria must have been introduced with the seeds.

We returned to a more detailed study of the problem only in 1944. The experiments were conducted with water cultures on peas, variety «De-Grace», in vessels of 6 litres capacity. In each vessel was introduced the full Hellriegel's mixture plus boron and manganese. The replication was a four-fold one. This mixture served as experimental control. On July 3rd 12 pea plantlets were planted into each vessel. Radium was introduced the next day, the concentrations being the same as in 1941, *viz.* $6 \cdot 10^{-10}$, $6 \cdot 10^{-9}$ and $6 \cdot 10^{-8}$ g of radium per one litre of the nutrient solution. All the vessels were inoculated on July 5th with a pure culture of root-nodule bacteria, *Bacterium radicicola*, which showed a strikingly rapid development in the presence of radioactive elements. Nodules could be noticed on the 5th day, and on the 20th day they were present in great numbers. In the control vessels which contained no radioactive elements, nodules were absent until the close of the growing period.

From Table 1 it may be seen that in the control vessels the average weight of the air-dry vegetative mass was 14.9 g per vessel, the weight of seeds was 6 g and the weight of the roots was 1.52 g per vessel. Root-nodules were entirely missing, in spite of the fact that the medium was inoculated with bacteria.

As a consequence of the introduction of radium at a dose of 10^{-10} g per litre of the solution the weight of the vegetative mass has increased by 34.3 per cent, the weight of the seeds by 58.3 per cent and the weight of the roots, by 35.5 per cent.

The weight of the nodules produced was 2.2 g per vessel.

A tenfold increase in the concentration of radium (10^{-9}g) did not increase the yield of vegetative mass and of roots as compared with the preceding dose, while the amount of seeds and the weight of root-nodules have noticeably increased.

A dose 100 times as great as the initial one (10^{-8} g) resulted in a certain decrease in the yield and weight of the root-nodules.

In our experiments it was important to know, however, not merely the yield, but, moreover, the ability of the bacteria to assimilate the atmospheric nitrogen in connexion with the presence of the radioactive elements.

Table 1

Effect of Radioactive Elements upon Yield of Pea and upon Development of Root-nodules

Experimental scheme	Average weight of air-dry parts						Weight of root-nodules in g/vessel
	stems		seeds		roots		
	g/vessel	%	g/vessel	%	g/vessel	%	
Control	14.90	100.0	6.00	100.0	1.52	100.0	0.00
Ra 10^{-10} g/l	20.01	134.3	9.50	158.3	2.06	135.5	2.22
Ra 10^{-9} g/l	20.01	134.3	9.90	165.0	1.90	126.5	2.52
Ra 10^{-8} g/l	19.88	133.5	8.71	145.0	2.01	133.2	1.62

With this aim in view we have determined the nitrogen content in the vegetative organs, in the seeds and in the roots of the plants, as well as the amount of nitrogen left behind in the nutrient mixture. The results are presented in Table 2.

Table 2

Experimental scheme	Nitrogen content to dry weight, %			Nitrogen content, g/vessel			Amount of residual nitrogen in nutrient solution, g/vessel	Nitrogen balance		Increase in nitrogen content, %
	stems	seeds	roots	stems	seeds	roots		introduced, g/vesssel	present, g/vessel	
1	2	3	4	5	6	7	8	9	10 (5+6+7+8)	11
Control . .	0.78	2.64	2.07	0.11	0.16	0.03	0.06	0.36	0 36	—
Ra 10^{-10} g/l	1.17	3.62	2.56	0.23	0.34	0.03	0.07	0.36	0.69	91.7
Ra 10^{-9} g/l .	1.27	3.24	2.69	0.25	0.32	0.05	0.08	0.36	0.70	94.4
Ra 10^{-8} g/l .	1.42	3.67	2.17	0.28	0.32	0.04	0.07	0.36	0.71	97.2

It may be observed from Table 2 that there occurred a considerable increase in the nitrogen content in the vegetative organs, in the seeds and roots of the plants grown in vessels containing radioactive elements.

Knowing the nitrogen content of the individual organs of the plants and the weight of the latter, as well as the amount of the nitrogen introduced and left behind in the nutrient solution, we may easily find the amount of nitrogen which was assimilated by the bacteria from the atmosphere under the influence of the root-nodule bacteria (Table 2). It appeared that the seeds, as well as the vegetative organs of the plants which had developed in vessels containing radioactive elements, contained nearly twice as much nitrogen as the controls. These results determine also the ultimate nitrogen balance. As seen from Table 2, the amount of nitrogen introduced into each vessel was 0.36 g, the amount contained in the plants being the same. No increase could take place, inasmuch as there was no root-nodule formation.

As the optimal dose of radium for the development of the plants and of the bacteria should be considered 10^{-10} g per litre of the nutrient solution. But, as shown by our experiments, the plants may well stand 10 and even 100 times greater doses of radium.

In contrast to our previous experiments we failed this time to observe any acceleration in the maturation of plants grown in vessels containing radio-

active elements where nodules were present on their roots. This phenomenon was explained in our experiment on sugar-beet, where it was shown that the plants which developed in the presence of radioactive elements were characterized by a more intense growth and more intense biochemical processes (especially at the initial period) which led to a considerably increased requirement in other nutrient stuffs, especially in nitrogen.

When the amount of nitrogen and of other nutrient materials is kept stable in the vessels, the best developed lateral leaves begin to turn yellow and gradually die off, which is not observed in the control vessels. In other words, artificial acceleration of the maturation of the plants takes place. When the plants receive an additional amount of nitrogen, whether in the form of fertilizers or by assimilation of free nitrogen by the root-nodule bacteria, no acceleration of maturation is observed, as may be seen from the present experiment. In such case there is a considerable increase in yield, as well as in the nitrogen content in the different parts of the plant, which is clearly shown by our experiments on peas.

Conclusions. 1. Root-nodule bacteria in general are extremely sensitive to the shortage in individual minor elements in the nutrient medium; radioactive elements occupy among them quite an outstanding place.

2. When radium and other radioactive elements are carefully eliminated from the nutrient medium, no root-nodules are formed on the roots of peas, even when the medium contains boron and manganese and is inoculated with root-nodule bacteria.

3. When radium is introduced at a dose of 10^{-10} g per 1 litre of the nutrient solution, root-nodule bacteria develop intensely on the roots of the young plants of peas and assimilate nearly twice as much molecular nitrogen of the air as the control, even when normal doses of nitrogen are contained in the nutrient solution.

The W. I. Vernadsky Laboratory
of Geochemical Problems.
Academy of Sciences of the USSR.

Received
28. IV. 1945.

REFERENCES

[1] A. A. Drobkov, Bull. Acad. Sci. URSS, sér. biol., No. 5 (1940).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

PLANT PHYSIOLOGY

EFFECT OF ALKALI UPON GERMINATING CAPACITY OF SEEDS OF KOK-SAGHYZ AND OF COMMON DANDELION

By O. A. PETROVA

(Communicated by A. A. Richter, Member of the Academy, 12. IV. 1945)

The seeds of the common non-rubber-yielding dandelion, *Taraxacum officinale* Wigg., differ from those of *Taraxacum kok-saghys* Rodin not merely in a number of morphological characters but, as shown by our studies, in their resistance to chemicals, namely to KOH and NaOH, as well as to concentrated sulphuric acid.

The treatment of seeds of kok-saghyz and of seeds of the common dandelion with a rather strong (up to 5 per cent) aquaeous solution of potassium or sodium hydrate during 60—90 minutes resulted in a considerable reduction of the germinating capacity of seeds of the common dandelion, while no reduction of the germinating capacity was produced in the case of kok-saghyz.

The experiment was arranged in the following way: samples consisting of 100 seeds each were placed in small glass-cups, and the alkaline solution was poured over them; after the seeds had remained in the solution for a definite length of time, the solution was poured out, the seeds were rinsed in water and transferred to Petri dishes for germination. Different concentrations of KOH and NaOH and different duration of treatment were tested. The effect of NaOH proved to be analogous to that of KOH, except for being somewhat stronger.

The keeping of seeds in a 5 per cent solution of KOH during half an hour failed to produce sufficiently clear results; the treatment during one hour and a half with the same solution resulted in a great reduction the germinating capacity of seeds of the common dandelion, whereas no influence upon the seeds of kok-saghyz could be recorded. 10 per cent solution of KOH caused a 100 per cent mortality of the seeds of the common dandelion and a considerable reduction in the germinating capacity of the seeds of kok-saghyz.

The treatment of the seeds with 5 per cent KOH solution was carried on in a fourfold replication. The following results were obtained:

	Kok-saghyz, alkali	Kok-saghyz, control	Dandelion, alkali	Dandelion, control
Average germinating capacity of the seeds, %	86	87	7	72

The germinating capacity of the seeds of kok-saghyz was the same in the control and after treatment with alkali. It should be added that the few seedlings of the common dandelion which had developed from the alkali-treated seeds proved abnormal, namely, they produced underdeveloped root-hairs, or

were entirely lacking root-hairs, and some of them showed dying root-tips. The seeds of kok-saghyz, on the contrary, after being treated with 5 per cent solution of KOH during one hour and a half produced mostly quite normal seedlings which differed in no way from the controls.

An analogous behaviour was observed in the case of seeds of the tetraploid *Taraxacum kok-saghys*, while the seeds of *Taraxacum serotinum* behaved in a way analogous to those of *Taraxacum officinale*.

The results of our experiments show the possibility of purifying the seeds of kok-saghyz from those of the weeds of the plantations of kok-saghyz, *viz.*, of the non-rubber-yelding *Taraxaci*.

There is ground to suggest that the different reaction of the seeds of kok-saghyz and of the non-rubber-bearing *Taraxaci* to the sodium and potassium hydrates is connected with some special features of the seed-coat in these species which, although closely related, differ in the intensity of the biochemical reactions taking place in the course of their development.

Received
12.IV.1945.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

ZOOLOGIE

POSITION SYSTÉMATIQUE DU CARACAL DU TURKESTAN (*FELIS CARACAL* MÜLL., 1776)

Par V. G. HEPTNER

(Présenté par I. I. Schmalhausen, de l'Académie, le 20. VI. 1945)

Le caracal est répandu en Afrique (à l'exclusion des régions boisées), en Arabie, en Syrie, en Palestine, dans la Perse du Sud, dans le Béloudjistan, dans le Sinda, dans le Pendjab et dans les régions centrales de l'Inde. Il existe également sans aucun doute en Afghanistan. En Asie Centrale il est connu dans le Turkménistan, qui forme ainsi la limite septentrionale de l'aire de l'habitat de l'espèce.

Dimensions des crânes de *Felis caracal michäelis Heptneri* (d'après les exemplaires conservés au Musée Zoologique de l'Université de Moscou)

Dimensions mesurées	Repetek, XI. 1935, N. V. Chibanov, S-9506	Bakhardok, 1940, V. G. Heptner, S-40194, Typel	Bakhardok, 1940, V. G. Heptner, S-40195	Bakhardok, 1940, V. G. Heptner, S-40196
Longueur maximale	—	142.0	136.8	127.0
Longueur condylobasale	131.9	129.4	125.0	116.5
Distance entre les arcades zygomatiques	92.6	93.4	93.4	83.2
Largeur de la région postorbitale derrière les processus supraorbitaux	30.4	29.0	27.9	31.7
Distance interorbitale	28.3	27.2	25.6	23.5
Distance entre les trous préorbitaux	38.3	38.0	37.7	35.5
Longueur de *nasalia*	35.5	33.2	32.0	—
Longueur alvéolaire de la rangée dentaire	44.7	44.6	42.1	40.5
Longueur de pm^4	17.0	17.2	16.8	16.7
Longueur maximale de la mandibule inférieure	100.2	99.0	94.9	86.9
Longueur alvéolaire de la rangée dentaire inférieure	48.7	48.4	46.5	44.7
Longueur de M_1	12.0	11.7	12.4	12.1

Le matériel des musées se rapportant à cette rare espèce de félins est très pauvre, ce qui explique principalement que l'on connaît très mal la variabilité géographique de l'espèce sur l'énorme territoire indiqué plus haut. De ce point de vue, l'étude des caracals du Turkestan n'a même pas été entreprise.

Les matériaux que nous avons rapportés du Turkménistan et qui sont conservés dans le Musée Zoologique de l'Université de Moscou, indiquent que les

caracals du Turkménistan ne sont pas identiques à la forme nominale qui se rencontre d'après les dernières recherches dans l'Inde, le Béloudjistan, le Sud de la Perse, l'Arabie (*t. typ.* de l'espèce) et également en Syrie et en Palestine.

On peut caractériser de la façon suivante la sous-espèce du Turkménistan.

Felis (*Caracal*) *caracal michaëlis* sbsp. nova

Type: No. S-40194. Sex ? Crâne. 1940. V. G. Heptner. Musée Zoologique de l'Université de Moscou.

Locus classicus: le puits de Bakhardok (Bokourdak) dans la région ouest du désert de Kara-Koum, à cent kilomètres au nord d'Achkhabad.

Diagnose: *F. c. michaëlis* se distingue de *F. c. caracal* par les dimensions plus grandes de son crâne.

Dimension du type: voir tableau ci-joint.

Distribution géographique: Turkménistan (région Transcaspienne).

Cette espèce est dédiée à M. V. Heptner.

Les caracals du Turkménistan dépassent considérablement par les dimensions de leur crâne la forme nominale. C'est ainsi que, par exemple, les animaux du Turkménistan dépassent par la longueur maximale du crâne les animaux du même âge de la race nominale (dont les dimensions sont données par R. J. Pocock[1]) de 17—21 mm, par la longueur condylobasale de 16.9—20.9 mm, par la distance entre les arcades zygomatiques de 7.4—11.4 mm, *etc.* En général, les animaux adultes de la forme nominale correspondent visiblement d'après leurs dimensions aux spécimens de l'espèce du Turkménistan n'ayant pas encore atteint leur plein développement.

L'auteur n'a pas de données sur les dimensions du corps du caracal du Turkménistan, mais les animaux nord-africains que l'auteur a pu observer en vie étaient de dimensions considérablement plus réduites que ceux du Turkestan.

Du point de vue systématique et zoo-géographique il est tout naturel qu'une race séparée de cette espèce, si répandue, se soit formée au Turkménistan, dans la zone la plus septentrionale de l'aire de l'habitat de l'espèce. Du même point de vue s'explique aisément l'apparition du principal caractère distinctif de cette race.

On peut vraisemblablement supposer que les caracals du Turkménistan ont été séparés (d'une manière probablement incomplète) de leurs congénères vivant plus au sud. La distribution du caracal dans le Turkménistan est liée principalement aux déserts sablonneux. L'auteur a pu montrer que l'on ne rencontre pas cet animal dans les régions montagneuses, ou que du moins il y est très rare. Et justement les plaines du Turkménistan sont limitées au sud par des massifs montagneux, ce qui fait que le caracal est plus commun dans les parties centrales du Turkménistan (désert du Kara-Koum) et se rencontre rarement dans le sud du pays, en direction de l'Afghanistan. En Turkménistan les caracals sont moins nombreux que les autres espèces de félins de petite taille, y compris le *Felis margarita*.

Musée Zoologique.
Université d'Etat de Moscou
M. V. Lomonossov

Manuscrit reçu le 20.VI.1945.

LITTÉRATURE CITÉE

[1] R. J. Pocock, The Fauna of British India including Ceylon and Burma, I, London, 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLIX, № 3

HELMINTHOLOGY

AEROBIC ENZYMIC SYSTEMS OF CERTAIN INTESTINAL HELMINTHS PARASITIC OF CATTLE AND SHEEP

By I. I. IVANOV and S. A. BURNASHEVA

(Communicated by K. I. Skrjabin, Member of the Academy, 12. III. 1945)

As well known, intestinal helminths are highly peculiar invertebrates which have become adapted to anaerobic conditions. It is of interest to note that these parasites obtain the energy necessary for their vital functions by way of glycogen break-up. The latter proceeds not so much under formation of lactic acid, as it is the case in animal tissues, but rather under liberation of various volatile fatty acids, *viz.* valeric acid, caproic acid, formic acid, butyric acid, as well as of dicarbonic acids, and of succinic acid, in particular ([1-5]) and others).

The chemical mechanism underlying this kind of fermentation has still remained rather obscure. Yet, intestinal and hepatic parasites, such as *Ascaris*, *Fasciola*, cestodes *Moniezia* and others, display a clearly pronounced respiratory function.

The present study is concerned with the existence in cestodes parasitic of cattle and sheep, *Thysaniezia ovilla*, both of a general respiratory function and of separate highly important enzymic components of the respiratory system, in particular, of indophenoloxidaze (the cytochromoxidaze) and succinoxidaze.

The cestodes which served as material for the experiments were supplied from the municipal slaughter-house. Respiration was measured with the aid of Warburg's apparatus, at a temperature of 37°C. Segments of cestodes were ground and reduced to suspension in NaCl-phosphate solution, with a pH = 7.4 and $\Delta t = 0.60$°C.

A set of preliminary experiments was also arranged, in order to make out whether the helminths investigated are able to discolour methylene-blue under anaerobic conditions in the presence of sodium succinate.

The results of the experiments on the respiration of both ground and integer segments of *Thysaniezia ovilla* are summed up in the table adjoined.

As may be inferred from the data tabulated, *Thysaniezia ovilla* cestodes are not only endowed with a clearly expressed respiratory function, but they also display the ability intensely to oxidize both paraphenylendiamine and sodium succinate, which can be seen from the sharp increase in their respiratory function when they are exposed to the influence of these substances. The rate of discolouring of methylene-blue treated with a paste prepared with ground and washed segments of these cestodes is also found to rise rapidly under anaerobic conditions in the presence of sodium succinate. Intestinal helminths have thus been found to preserve at the tapeworm stage of development, in spite of their anaerobic mode of life*, the most important aerobic enzymic

* Intestinal gases contain scarcely any oxygen, if at all([6,7]).

Respiratory Capacity of *Thysaniezia ovilla* Treated with Paraphenylendiamine and Sodium Succinate

Breathing tissue	Duration of experiment in min.	Absorption of O_2 in mm^3 per 1 g of tissue (raw weight)							
		in NaCl phosphate solution		with sodium succinate (0.5%)		with paraphenylenediamine (0.5%)		With sodium fumarate present in the air	With sodium malate present in the air
		in O_2	in the air	in O_2	in the air	in O_2	in the air		
Ground cestode segments (not washed)	45	—	30.2	—	83.2	—	57.2	—	—
	45	—	72.5	—	163	—	113	—	—
	30	—	50.4	—	141	—	125.7	—	—
	30	70.6	59	151	141	122	88.9	—	—
	30	—	78.1	—	117.2	—	—	30.5	24.7
	30	—	75	—	132.9	—	—	40	42.5
	30	—	66.8	—	127.3	—	120.4	30.5	38.4
Ground cestode segments washed in water	45	—	10.7	—	22	—	25.4	—	—
	45	—	5.5	—	39	—	17.1	—	—
	45	—	17.8	—	111.7	—	63.5	—	—
	30	—	15.7	—	95.8	—	46	—	—
	30	25.3	16.4	82.5	92	60.9	50.8	—	—
Unground cestode segments	45	—	15.24	—	118.1	—	—	—	—
	45	—	—	—	137.3	—	—	—	—
	45	—	78.3	—	135.7	—	195.4	—	—
	30	56.2	60.5	103.9	103.3	118.5	93.9	—	—
	30	90.4	63	133.7	103.9	124.5	90.2	—	—
	30	—	60.5	—	—	—	—	25.4	48.8
	30	—	—	—	138.3	—	—	95.2	83.2

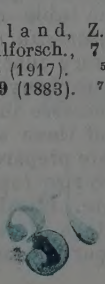
systems peculiar of animal tissues; to these belongs the system serving to oxidize paraphenylendiamine, the cytochromoxidaze (indophenoloxidaze), and the one serving for the oxidation of succinic acid (succindehydraze + cytochrome).

The above data are of obvious interest in general physiology of helminths; they will also greatly contribute to the knowledge of the evolution of intestinal parasites. In the author's opinion, they may well be regarded as one of the proofs of the origin of the recent intestinal parasites from free-living animal forms.

In concluding we esteem it our agreeable duty to express our sincere thanks to K. I. Skrjabin, Member of the Academy, for the continuous interest he took in the present work, and for many a valuable advices made in its course.

Biochemical Laboratory.
Kirghizian Branch of the Academy of Sciences of the USSR.
Kirghizian State Medical Institute.

Received
12. III. 1945.

REFERENCES

[1] E. Weinland, Z. Biol., **41** (1900); **42** (1901); **43** (1902); **45** (1903/1904). [2] Kemnitz, Arch. Zellforsch., **7** (1912). [3] Fischer, Bioch. Z., **144** (1924). [4] Schulte, Pflüg. Arch., **166** (1917). [5] v. Brand, Ergebn. d. Biologie, **10**, 37 (1934). [6] Tappeiner, Z. Biol., **19** (1883). [7] v. Brand und Weise, Z. vergl. Physiol., **13** (1932).